Universidade de Brasília (UnB)
Faculdade de Tecnologia (FT)
Departamento de Engenharia Elétrica (ENE)

Course: Image Processing
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Semester: 2017.1

LIST 02

Submission Date: 13/04/2017 (Cut-off: 20/02/2017)

Question 1: Prove that both the Discrete and Continuous Fourier Transform are linear operations.

Question 2: Prove the following properties of the 2D DFT:

P1:

$$f(x,y)e^{j2\pi(u_0x/M+v_0y/N)} \Longleftrightarrow F(u-u_0,v-v_0)$$

P2:

$$f(x-x_0,y-y_0) \Longleftrightarrow F(u,v)e^{j2\pi(x_0u/M+y_0u/N)}$$

Question 3: Show that the DFT $f(x,y) = \sin(2\pi u_0 x + 2\pi v_0 y)$ is given by:

$$F(u,v) = \frac{j}{2}\left[\delta(u+Mu_0,v+Nv_0) - \delta(u-Mu_0,v-Nv_0)\right].$$

Question 4: Prove all properties shown in the Table shown in Fig.1.

Question 5: Prove that the periodicity properties (in the spatial and frequency domain) of the 2D DFT.

Question 6: Prove the convolution theorem (2D DFT).

Question 7: Prove the differentiation theorem (2D DFT).

	Spatial Domain†		Frequency Domain†
1)	$f(x, y)$ real	$\Leftrightarrow$	$F^*(u, v) = F(-u, -v)$
2)	$f(x, y)$ imaginary	$\Leftrightarrow$	$F^*(-u, -v) = -F(u, v)$
3)	$f(x, y)$ real	$\Leftrightarrow$	$R(u, v)$ even; $I(u, v)$ odd
4)	$f(x, y)$ imaginary	$\Leftrightarrow$	$R(u, v)$ odd; $I(u, v)$ even
5)	$f(-x, -y)$ real	$\Leftrightarrow$	$F^*(u, v)$ complex
6)	$f(-x, -y)$ complex	$\Leftrightarrow$	$F(-u, -v)$ complex
7)	$f^*(x, y)$ complex	$\Leftrightarrow$	$F^*(-u - v)$ complex
8)	$f(x, y)$ real and even	$\Leftrightarrow$	$F(u, v)$ real and even
9)	$f(x, y)$ real and odd	$\Leftrightarrow$	$F(u, v)$ imaginary and odd
10)	$f(x, y)$ imaginary and even	$\Leftrightarrow$	$F(u, v)$ imaginary and even
11)	$f(x, y)$ imaginary and odd	$\Leftrightarrow$	$F(u, v)$ real and odd
12)	$f(x, y)$ complex and even	$\Leftrightarrow$	$F(u, v)$ complex and even
13)	$f(x, y)$ complex and odd	$\Leftrightarrow$	$F(u, v)$ complex and odd

†Recall that x, y, u, and v are *discrete* (integer) variables, with x and u in the range $[0, M - 1]$, and y, and v in the range $[0, N - 1]$. To say that a complex function is *even* means that its real *and* imaginary parts are even, and similarly for an odd complex function.

Figura 1: Table of the properties of the 2D DFT.