Image Processing

Restoration and Reconstruction

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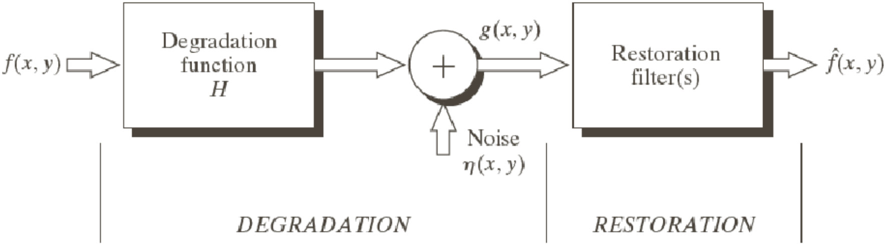
4 de Abril de 2017

Class 07: Restoration and Reconstruction

Overview

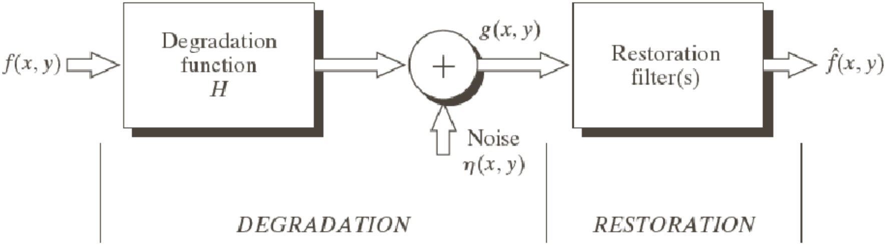
- Types of Noise
- Restoration Techniques
- Linear Degradations
- Estimation of the Degradation Function
- Inverse Filtering

Model of the Degradation and Restoration Processses



$$g(x, y) = h(x, y) * f(x, y) + \eta(x, y)$$

Model of the Degradation and Restoration Processses



$$G(u, v) = H(u, v) \cdot F(u, v) + N(u, v)$$

Noise

- Gaussian Noise:

$$p(z) = \frac{1}{\sqrt{2\pi\sigma}} e^{-(z-\mu)^2/2\sigma^2}$$

- Rayleigh Noise:

$$p(z) = \begin{cases} \frac{2}{b}(z-1)e^{-(z-a)^2/b}, & \text{for } z \geq a \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

- Erlang Noise:

$$p(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!} e^{-az}, & \text{for } z \geq 0 \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

Noise

- Exponential Noise:

$$p(z) = \begin{cases} ae^{-az}, & \text{for } z \geq 0 \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

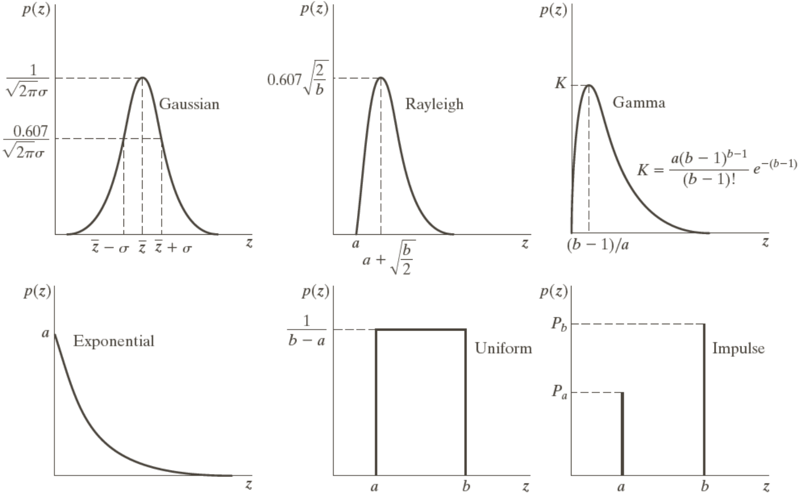
- Uniform Noise:

$$p(z) = \begin{cases} \frac{1}{(b-a)}, & \text{for } b \leq z \leq a \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

- Impulsive Noise:

$$p(z) = \begin{cases} P_a, & \text{for } z = a \\ P_b, & \text{for } z = b \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

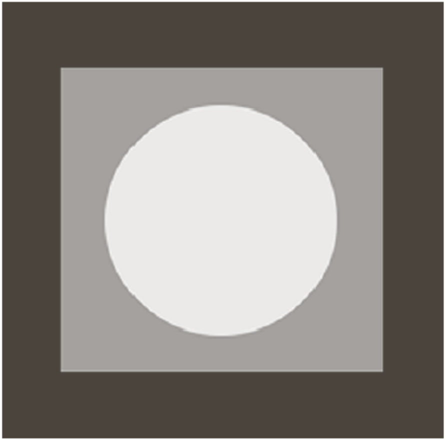
Probability Density Functions of Different Types of Noise



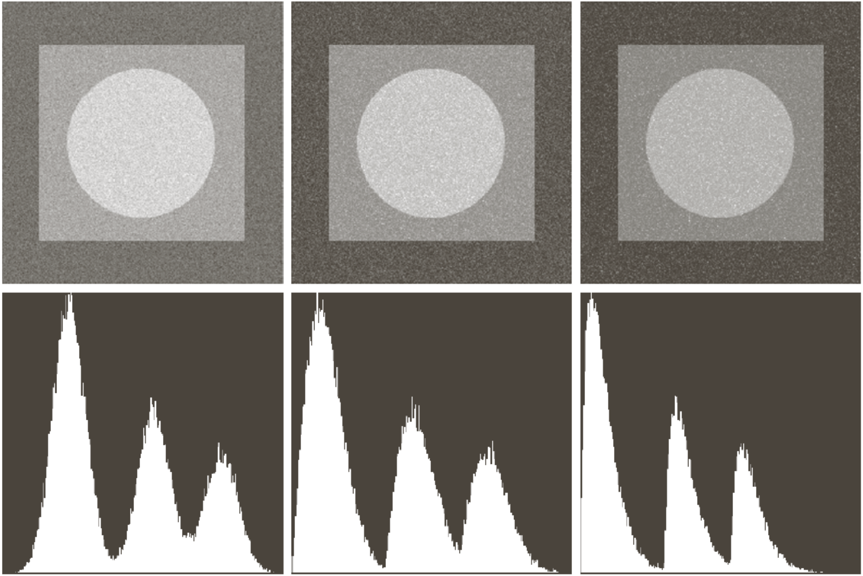
a b c
d e f

FIGURE 5.2 Some important probability density functions.

Examples

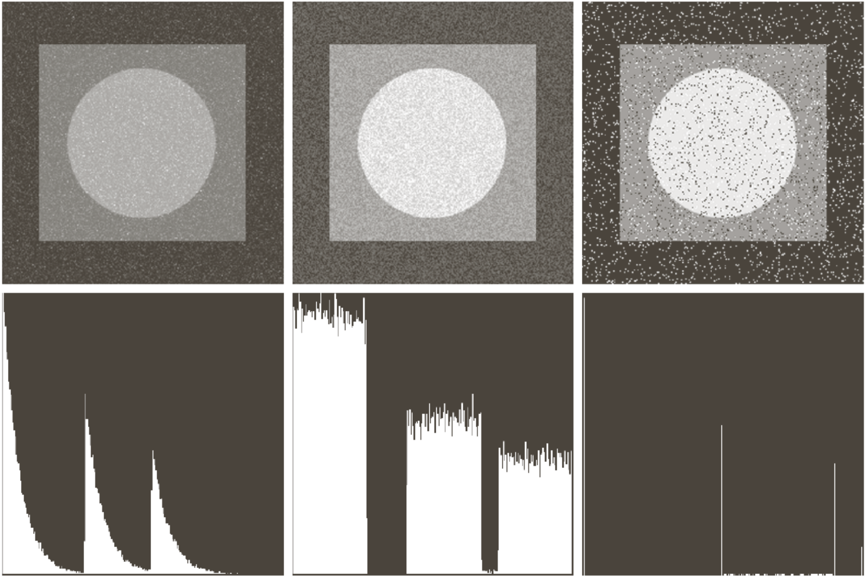


Examples



Gaussian Rayleigh Gamma

Examples

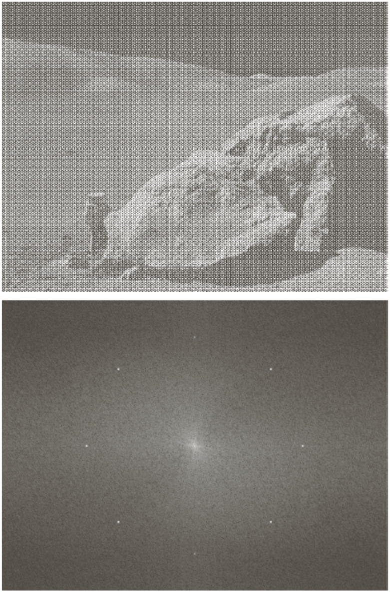


Exponential Uniform Salt & Pepper

Ways to Extract Noise

- spectrum
- spatial domain
- analysing the noise pattern (known source)
- analysing image regions

Periodic Noise



a
b

FIGURE 5.5 (a) Image corrupted by sinusoidal noise. (b) Spectrum (each pair of conjugate impulses corresponds to one sine wave). (Original image courtesy of NASA.)

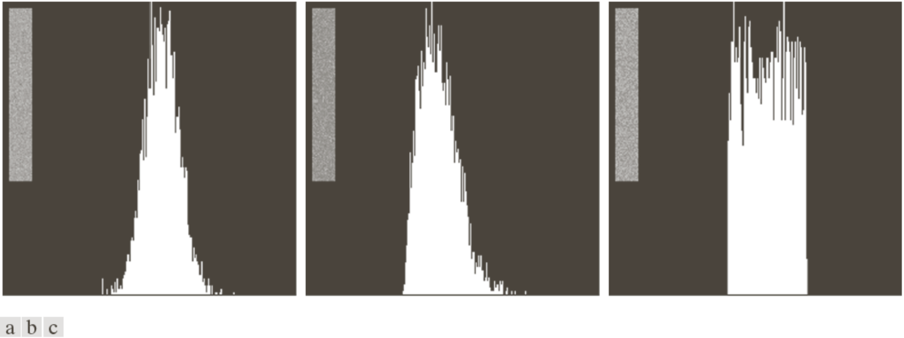


FIGURE 5.6 Histograms computed using small strips (shown as inserts) from (a) the Gaussian, (b) the Rayleigh, and (c) the uniform noisy images in Fig. 5.4.

Noise

Filtering

- frequency domain
- spatial domain

Spatial Filtering

$$g(x, y) = f(x, y) + \eta(x, y)$$

$$G(u, v) = F(u, v) + N(u, v)$$

- Additive Noise – Spatial Filtering;
- Spatial Filters – as shown in previous chapters.

- Artihmetic Average:

$$\hat{f}(x,y) = \frac{1}{mn}\sum_{(s,t)\in S_{x,y}} g(s,t)$$

- Median:

$$\hat{f}(x,y) = \text{Median}_{(s,t)\in S_{x,y}}\{g(s,t)\}$$

- Geometric Averages:

$$\hat{f}(x,y) = \left[\prod_{(s,t)\in S_{x,y}} g(s,t)\right]^{mn}$$

Spatial Filtering

- Harmonic Mean Filter:

$$\hat{f}(x,y) = \frac{mn}{\sum_{(s,t)\in S_{x,y}} \frac{1}{g(s,t)}}$$

- Contraharmonic Mean Filter:

$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{x,y}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{x,y}} g(s,t)^{Q}}$$

- Max and Min Filters:

$$\hat{f}(x,y) = \max_{(s,t)\in S_{x,y}} \{g(s,t)\}$$

$$\hat{f}(x,y) = \min_{(s,t)\in S_{x,y}} \{g(s,t)\}$$

Spatial Filtering

- Midpoint Filter:

$$\hat{f}(x,y) = \frac{1}{2}\left[\max_{(s,t)\in S_{x,y}}\{g(s,t)\} + \min_{(s,t)\in S_{x,y}}\{g(s,t)\}\right]$$

- Alpha-trimmed Filter:

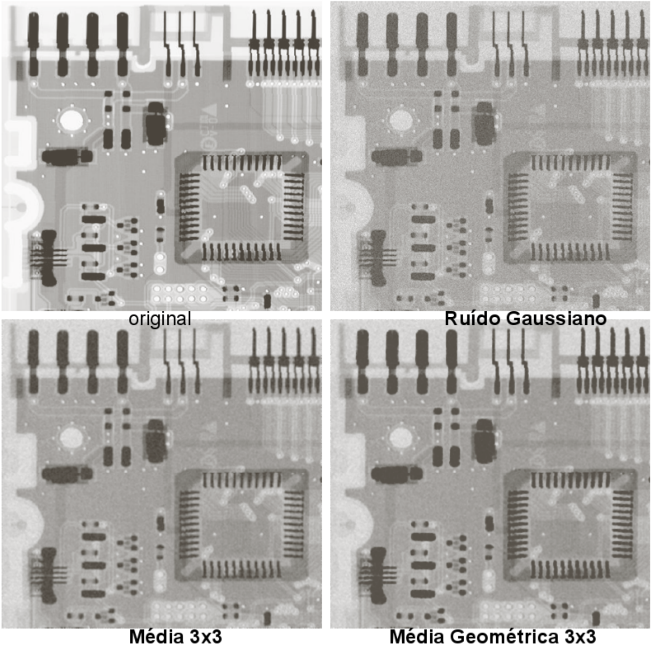
$$\hat{f}(x,y) = \frac{1}{mn-d}\sum_{(s,t)\in S_{x,y}} g(s,t)$$

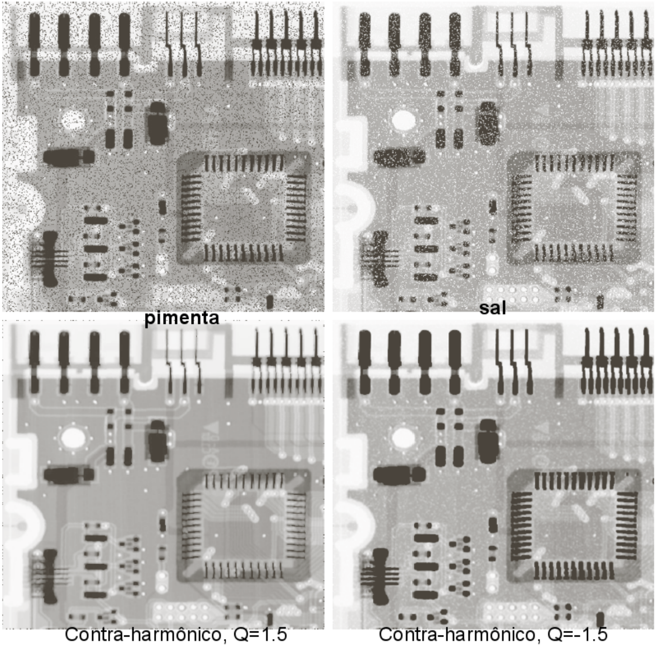
Delete the smaller $d/2$ and the highest $d/2$ values in the neighborhood $S_{x,y}$.

a	b
c	d

FIGURE 5.7 (a) X-ray image. (b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size 3 × 3. (d) Result of filtering with a geometric mean filter of the same size. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

O filtro média geométrica causou menos borrado.



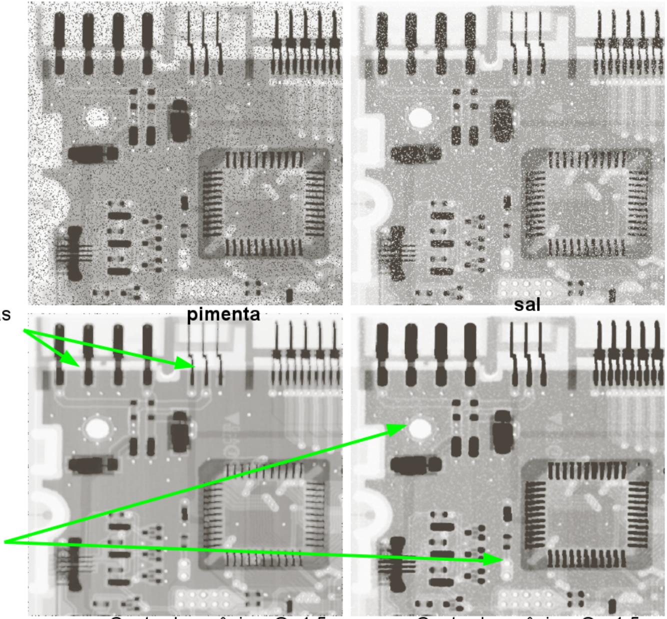


a b
c d

FIGURE 5.8
(a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability. (c) Result of filtering (a) with a 3 × 3 contra-harmonic filter of order 1.5. (d) Result of filtering (b) with $Q = -1.5$.

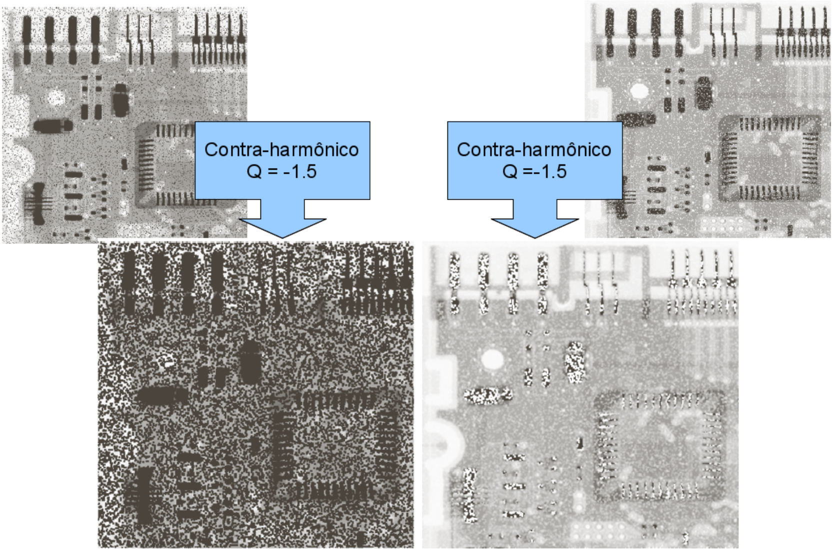
Q>0, afinamento
e borramento
das áreas escuras

Q<0, afinamento
e borramento
das áreas claras



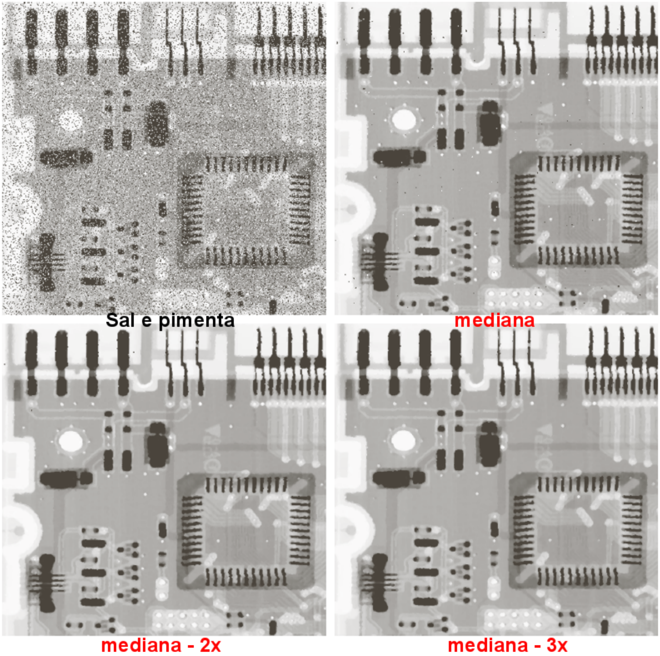
Contra-harmônico, Q=1.5

Contra-harmônico, Q=-1.5

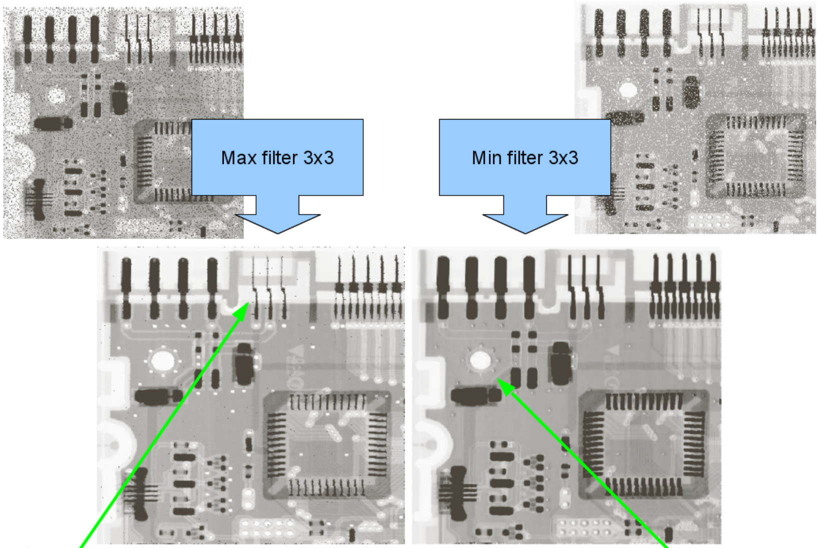

Contra-harmônico
Q = -1.5
Contra-harmônico
Q =-1.5

a	b
c	d

FIGURE 5.10 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.1$. (b) Result of one pass with a median filter of size 3×3. (c) Result of processing (b) with this filter. (d) Result of processing (c) with the same filter.

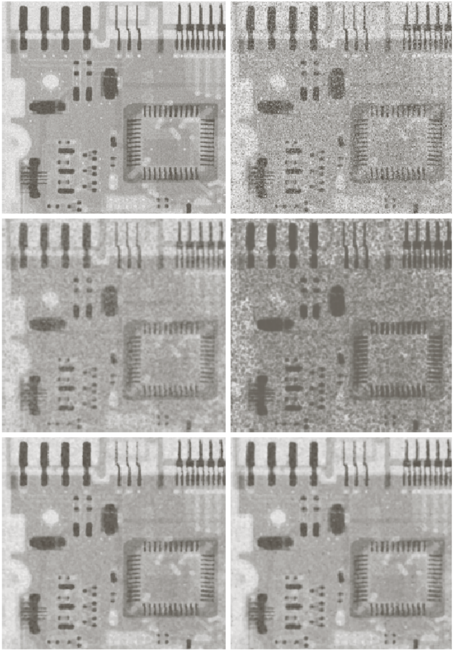


Filtrando com max e min



Detalhes em preto diminuíram.

Detalhes em branco diminuíram.



Ruído uniforme

Ruído uniforme +
pimenta-e-sal

Filtro média

Filtro média geométrica

Filtro mediana

Filtro alpha-podado
d=5

Adaptive Filter

- Changes for each region - adapting to the image statistics
- Higher computational cost
  - Average - average intensity level
  - Variance - average contrast
- Region Sxy:
  - g(x,y) value, variance, average, noise

Adaptive Filter

$$g(x,y) = f(x,y) + \eta(x,y)$$

- If $\sigma_\eta^2 = 0$, the filter returns $g(x,y)$
- If the local variance σ_L^2 is higher than σ_η^2, the filter returns $g(x,y)$ (borders)
- If the variances are similar, an arithmetic average filter must be applied:

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_\eta^2}{\sigma_L^2}\left[g(x,y) - m_L\right], \quad \sigma_\eta^2 \ll \sigma_L^2$$

Adaptive Filter (Median)

z_{min} = minimum intensity value in $S_{x,y}$
z_{max} = maximum intensity value in $S_{x,y}$
z_{med} = median intensity value in $S_{x,y}$
z_{xy} = intensity value of (x, y) coordinates
S_{max} = maximum size for $S_{x,y}$

Level A:

$A_1 = z_{med} - z_{min}$
$A_2 = z_{med} - z_{max}$
IF $A_1 > 0$ e $A_2 < 0$, go to level B
ELSE increase the window size
IF window size $\leq S_{max}$ repeat level A
ELSE output $= z_{xy}$

Level B:

$B_1 = z_{xy} - z_{min}$
$B_2 = z_{xy} - z_{max}$
IF $B_1 > 0$ e $B_2 < 0$, output $= z_{xy}$
ELSE output $= z_{med}$

a	b
c	d

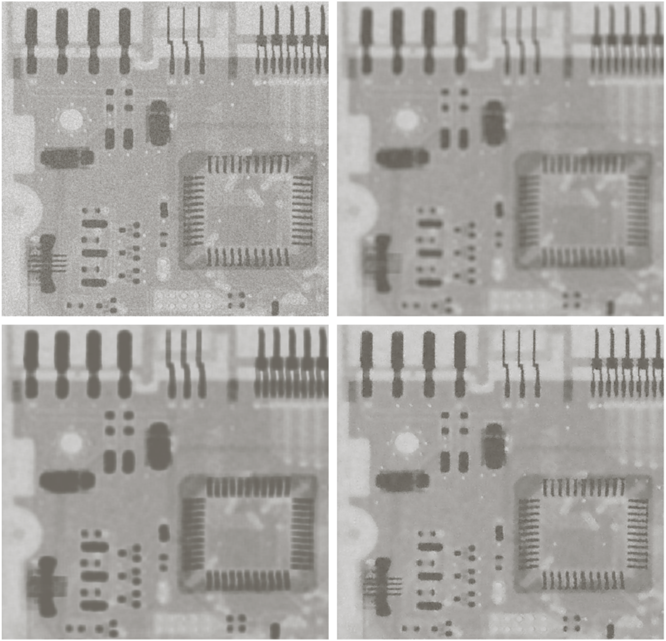
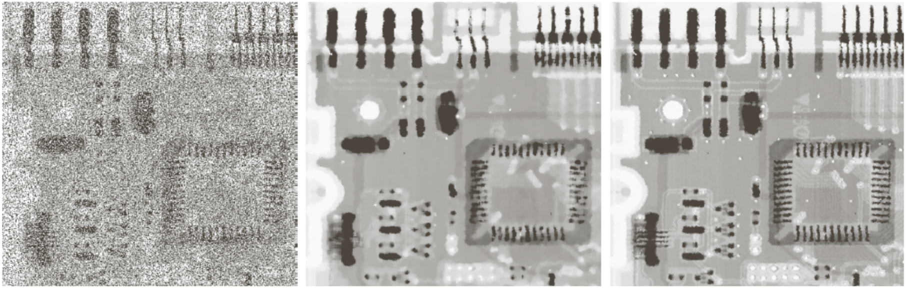


FIGURE 5.13 (a) Image corrupted by additive Gaussian noise of zero mean and variance 1000. (b) Result of arithmetic mean filtering. (c) Result of geometric mean filtering. (d) Result of adaptive noise reduction filtering. All filters were of size 7×7.



a b c

FIGURE 5.14 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.25$. (b) Result of filtering with a 7×7 median filter. (c) Result of adaptive median filtering with $S_{\max} = 7$.

Optimum Notch Filter

$$g(x, y) = f(x, y) + \eta(x, y)$$
$$N(u, v) = H_{NP}(u, v) \cdot G(u, v)$$

Optimum Notch Filter

$$g(x, y) = f(x, y) + \eta(x, y)$$
$$N(u, v) = H_{NP}(u, v) \cdot G(u, v)$$

$$\eta(x, y) = TF^{-1}\left(H_{NP}(u, v) \cdot G(u, v)\right)$$

Optimum Notch Filter

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$$\hat{f}(x,y) = f(x,y) - w(x,y)\eta(x,y)$$

Choose $w(x,y)$ to minimize the variance of the estimative

$$g(x,y) = f(x,y) + \eta(x,y)$$
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$$\eta(x,y) = TF^{-1}\left(H_{NP}(u,v) \cdot G(u,v)\right)$$

$$\hat{f}(x,y) = f(x,y) - w(x,y)\eta(x,y)$$

Choose $w(x,y)$ to minimize the variance of the estimative

$$\sigma^2(x,y) = \frac{1}{(2a+1)(2b+1)} \sum_{s=-a}^{a} \sum_{t=-b}^{b} \left[\hat{f}(x+s,y+t) - \overline{f}(x,y)\right]^2$$

$$\overline{f(x,y)} = \frac{1}{(2a+1)(2b+1)} \sum_{s=-a}^{a} \sum_{t=-b}^{b} \hat{f}(x+s,y+t)$$

$$w(x+s,y+t) = w(x,y)$$

$$\overline{w(x+s,y+t)\eta(x,y)} = w(x+s,y+t)\overline{\eta(x,y)}$$

Optimum Notch Filter

To minimize $\sigma^2(x,y)$, solve:

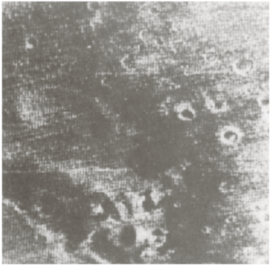
$$\frac{\sigma^2(x,y)}{dw(x,y)} = 0$$

The result is:

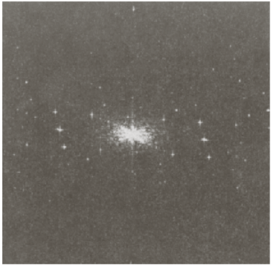
$$w(x,y) = \frac{\overline{g(x,y)\cdot\eta(x,y)} - \overline{g(x,y)}\cdot\overline{\eta(x,y)}}{\overline{\eta^2(x,y)} - \overline{\eta(x,y)}^2}$$

So, it suffices to use the above result to obtain $\hat{f}(x,y)$:

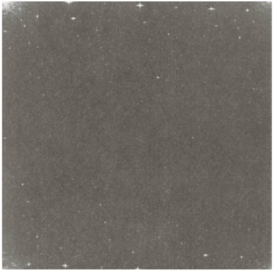
$$\hat{f}(x,y) = f(x,y) - w(x,y)\eta(x,y)$$



original



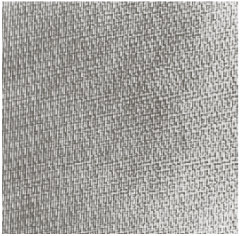
Espectro deslocado



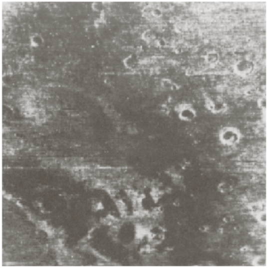
Espectro sem deslocamento

a=b=15

Espectro do Ruído



Ruído no domínio espacial



Processada

Linear and Time-Invariant Degradations

$$g(x,y) = H[f(x,y)] + \eta(x,y)$$

$$\boxed{\text{Se } \eta(x,y) = 0 \Longrightarrow g(x,y) = H[f(x,y)]}$$

Since the system is linear:

$$\boxed{H[a \cdot f_1(x,y) + b \cdot f_2(x,y)] = a \cdot H[f_1(x,y)] + b \cdot H[f_2(x,y)]}$$

And time-invariant:

$$\boxed{H[f(x-\alpha, y-\beta)] = g(x-\alpha, y-\beta)}$$

Linear and Time-Invariant Degradations

$$g(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)h(x-\alpha,y-\beta)d\alpha d\beta + \eta(x,y)$$

If $\eta(x,y) = 0$

$$g(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)h(x-\alpha,y-\beta)d\alpha d\beta$$

$$\boxed{G(u,v) = H(u,v)F(u,v) + N(u,v)}$$

$$\boxed{g(x,y) = h(x,y)h(x,y) + \eta(x,y)}$$

Degradation Estimation

- Observation:

$$H_s(u,v) = \frac{G_s(u,v)}{\hat{F}(u,v)}$$

- Experimentation:

$$H_s(u,v) = \frac{G(u,v)}{A}$$

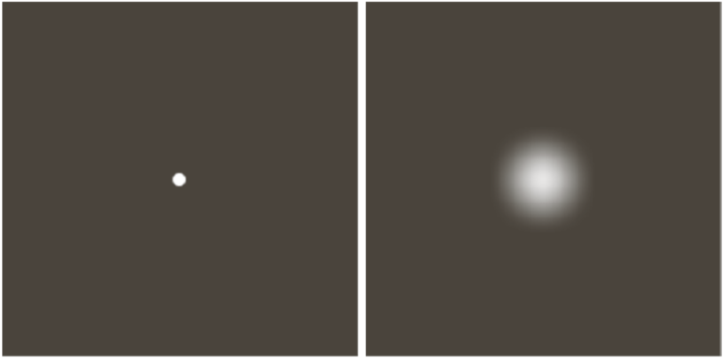
- Mathematical Modeling:

$$H(u,v) = e^{-k(u^2+v^2)^{5/6}}$$

degradation model proposed by Hufnagel and Stanley (1964), based on atmospheric turbulence.

a b

FIGURE 5.24 Degradation estimation by impulse characterization. (a) An impulse of light (shown magnified). (b) Imaged (degraded) impulse.



impulso

Imagem degradada peloimpulso

Atmospheric Turbulence model:

$$H(u,v) = e^{-k(u^2+v^2)^{5/6}}$$

k=0,0025

k=0,001

k=0,00025

Blur Model (Motion BLur)

$$g(x, y) = \int_0^T f(x - x_0(t), y - y_o(t))dt$$

Blur Model (Motion BLur)

$$g(x,y) = \int_0^T f(x - x_0(t), y - y_o(t))dt$$

$$\begin{aligned} G(u,v) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)e^{-j2\pi(ux+uy)}dxdy \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left[\int_0^T f\left[x - x_0(t), y - y_o(t)\right]dt\right] e^{-j2\pi(ux+uy)}dxdy \end{aligned}$$

Blur Model (Motion BLur)

$$g(x, y) = \int_0^T f(x - x_0(t), y - y_o(t))dt$$

$$\begin{aligned} G(u, v) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x, y) e^{-j2\pi(ux+uy)} dxdy \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[\int_0^T f\left[x - x_0(t), y - y_o(t)\right] dt \right] e^{-j2\pi(ux+uy)} dxdy \end{aligned}$$

$$G(u, v) = \int_0^T \left[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f\left[x - x_0(t), y - y_o(t)\right] e^{-j2\pi(ux+uy)} dxdy \right] dt$$

$$\begin{aligned} G(u, v) &= \int_0^T F(u, v) e^{-j2\pi[ux_o(t)+uy_0(t)]} dt \\ &= F(u, v) \int_0^T e^{-j2\pi[ux_o(t)+uy_0(t)]} dt \end{aligned}$$

Blur Model (Motion BLur)

$$G(u,v) = \int_0^T \left[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f\left[x - x_0(t), y - y_o(t)\right] e^{-j2\pi(ux+uy)} dxdy \right] dt$$

$$G(u,v) = \int_0^T F(u,v) e^{-j2\pi[ux_o(t)+uy_0(t)]} dt$$

$$= F(u,v) \int_0^T e^{-j2\pi[ux_o(t)+uy_0(t)]} dt$$

$$G(u,v) = \int_0^T \left[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f\left[x - x_0(t), y - y_o(t)\right] e^{-j2\pi(ux+uy)} dxdy \right] dt$$

$$G(u,v) = \int_0^T F(u,v) e^{-j2\pi[ux_o(t)+uy_0(t)]} dt$$

$$= F(u,v) \int_0^T e^{-j2\pi[ux_o(t)+uy_0(t)]} dt$$

$$\boxed{H(u,v) = \int_0^T e^{-j2\pi[ux_o(t)+uy_0(t)]} dt}$$

$$G(u,v) = \int_0^T \left[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f\left[x - x_0(t), y - y_o(t)\right] e^{-j2\pi(ux+uy)} dxdy \right] dt$$

$$G(u,v) = \int_0^T F(u,v) e^{-j2\pi[ux_o(t)+uy_0(t)]} dt$$

$$= F(u,v) \int_0^T e^{-j2\pi[ux_o(t)+uy_0(t)]} dt$$

$$\boxed{H(u,v) = \int_0^T e^{-j2\pi[ux_o(t)+uy_0(t)]} dt}$$

$$\boxed{G(u,v) = H(u,v)F(u,v)}$$

Example

$$H(u,v) = \int_0^T e^{-j2\pi[ux_o(t)+uy_0(t)]}dt$$

Example

$$H(u,v) = \int_0^T e^{-j2\pi[ux_o(t)+uy_0(t)]}dt$$

- Movement in the x direction: $x_0(t) = \frac{a \cdot t}{T}$

Example

$$\boxed{H(u,v) = \int_0^T e^{-j2\pi[ux_o(t)+uy_0(t)]}dt}$$

- Movement in the x direction: $x_0(t) = \frac{a \cdot t}{T}$

$$\begin{aligned} H(u,v) &= \int_0^T e^{-j2\pi u x_o(t)} dt \\ &= \int_0^T e^{\frac{-j2\pi u \cdot at}{T}} dt \\ &= \frac{T}{\pi ua} \sin(\pi ua) e^{-j\pi ua} \end{aligned}$$

Example

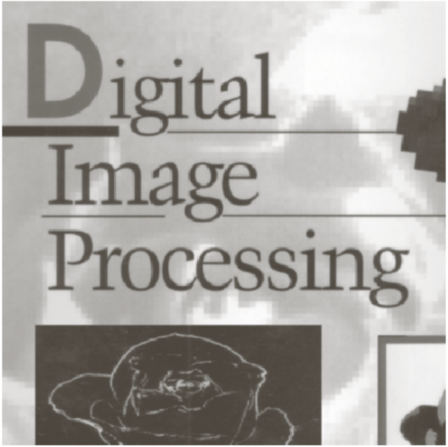
$$H(u,v) = \int_0^T e^{-j2\pi[ux_o(t)+uy_0(t)]}dt$$

- Movement in the x direction: $x_0(t) = \frac{a \cdot t}{T}$

$$\begin{aligned} H(u,v) &= \int_0^T e^{-j2\pi u x_o(t)} dt \\ &= \int_0^T e^{\frac{-j2\pi u \cdot at}{T}} dt \\ &= \frac{T}{\pi u a} \sin(\pi u a) e^{-j\pi u a} \end{aligned}$$

Or, more generically: (blur in both directions)

$$H(u,v) = \frac{T}{\pi(ua+vb)} \sin(\pi(ua+vb)) e^{-j\pi(ua+vb)}$$



$$H(u, v) = \frac{T}{\pi(ua + vb)} \sin[\pi(ua + vb)] e^{-j\pi(ua+vb)}$$

Borrando a imagem
com a=b=0,1
T=1

Inverse Filtering

- If we know the degradation, can we directly restore the image?

Inverse Filtering

- If we know the degradation, can we directly restore the image?

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}$$

$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

$$\hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

Inverse Filtering

- If we know the degradation, can we directly restore the image?

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- Even if we know the degradation, we cannot recover the signal completely ...
- And if $H(u,v)$ has zero values?

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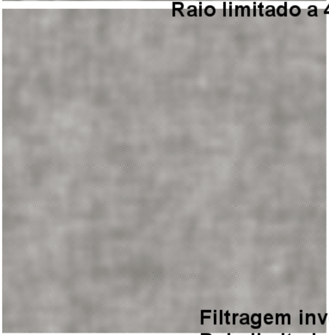
- Even if we know the degradation, we cannot recover the signal completely ...
- And if $H(u,v)$ has zero values?
- Option: Limit the values of the filter around (0,0).

Filtragem inversa

Filtragem inversa
Raio limitado a 40

Filtragem inversa
Raio limitado a 70



Filtragem inversa
Raio limitado a 85

Wiener Filter

$$e^2 = E\left[\left(f - \hat{f}\right)^2\right] \quad (6)$$

$$\begin{aligned}\hat{F}(u,v) &= \left[\frac{H^*(u,v)S_f(u,v)}{S_f(u,v)|H(u,v)|^2 + S_\eta(u,v)}\right]G(u,v)\\ &= \left[\frac{H^*(u,v)}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)}\right]G(u,v)\\ &= \left[\frac{1}{H(u,v)}\frac{|H(u,v)|^2}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)}\right]G(u,v)\end{aligned}$$

$H(u,v)$ = degradation function

$H^*(u,v)$ = complex conjugate of $H(u,v)$

$|H(u,v)|^2 = H^*(u,v)H(u,v)$

$S_\eta(u,v) = |N(u,v)|^2$ = power spectrum of the noise [see Eq. (4.2-20)]

$S_f(u,v) = |F(u,v)|^2$ = power spectrum of the undegraded image.

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + K} \right] G(u,v)$$

a b c

FIGURE 5.28 Comparison of inverse and Wiener filtering. (a) Result of full inverse filtering of Fig. 5.25(b). (b) Radially limited inverse filter result. (c) Wiener filter result.

Wiener Filter

corrompida | Filtragem inversa | Wiener

Borrado de movimento
+ ruído Gaussiano
Média = 0, var = 650

var é diminuída de 1
ordem de magnitude

var é diminuída de 5
ordens de magnitude