Processamento de Imagens

Introdução

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Aula 05: Processamento no Domínio da Frequência

Série de Fourier

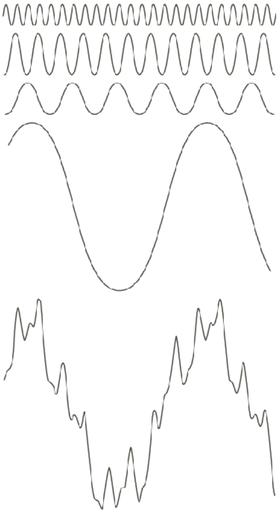


FIGURE 4.1 The function at the bottom is the sum of the four functions above it. Fourier's idea in 1807 that periodic functions could be represented as a weighted sum of sines and cosines was met with skepticism.

Série de Fourier

Análise

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{j\frac{2\pi nt}{T}}$$

Síntese

$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t) e^{-j\frac{2\pi nt}{T}} dt$$

Transformada de Fourier Contínua 1-D

Transformada de Fourier Contínua

Análise

$$F\left\{f(t)\right\} = F(\mu) = \int_{-\infty}^{\infty} f(t) e^{-j2\pi\mu t} dt$$

Síntese

$$f(t) = \int_{-\infty}^{\infty} F(\mu) e^{j2\pi\mu t} d\mu$$

Exemplo 1: Calcule a TF de $f(t)$

$$f(t) = \begin{cases} 0 & \text{se } |t| > W/2 \\ A & \text{se } |t| \leq W/2 \end{cases}$$

$$\begin{aligned} F(\mu) &= \int_{-\infty}^{\infty} f(t) e^{-j2\pi\mu t} dt = \int_{-W/2}^{W/2} A e^{-j2\pi\mu t} dt \\ &= \frac{-A}{j2\pi\mu} \left[e^{-j2\pi\mu t} \right]_{W/2}^{W/2} = \frac{-A}{j2\pi\mu} \left[e^{-j\pi\mu W} - e^{j\pi\mu W} \right] \\ &= AW \frac{\sin(\pi\mu W)}{\pi\mu W} = AW \text{sinc}(\mu W) \end{aligned}$$

Exemplo 1: Calcule a TF de $f(t)$

$$f(t) = \begin{cases} 0 & \text{se } |t| > W/2 \\ A & \text{se } |t| \leq W/2 \end{cases}$$

$$F(\mu) = AW\text{sinc}(\mu W)$$

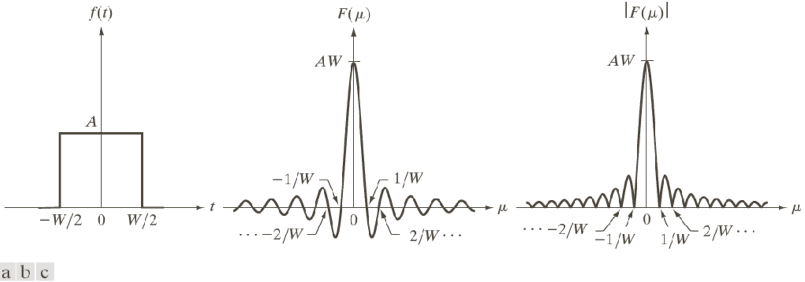


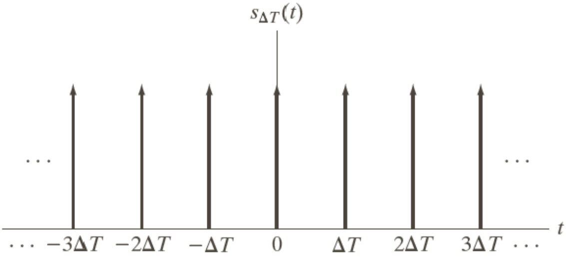
FIGURE 4.4 (a) A simple function; (b) its Fourier transform; and (c) the spectrum. All functions extend to infinity in both directions.

Exemplo 2: Calcule a TF de $f(t) = \delta(t)$

$$F(\mu) = \int_{-\infty}^{\infty} f(t) e^{-j2\pi\mu t} dt$$
$$= \int_{-\infty}^{\infty} \delta(t) e^{-j2\pi\mu t} dt = e^{-j2\pi\mu 0} = 1$$

Exemplo 3: Calcule a TF de $f(t) = \delta(t - t_0)$

$$F(\mu) = \int_{-\infty}^{\infty} f(t) e^{-j2\pi\mu t} dt$$
$$= \int_{-\infty}^{\infty} \delta(t - t_0) e^{-j2\pi\mu t} dt = e^{-j2\pi\mu t_0}$$
$$= \cos(2\pi\mu t_0) - j\sin(2\pi\mu t_0)$$



Exemplo 4: Calcule a TF de um *trem de impulsos*

$$s_{\Delta T}(t) = \sum_{n=-\infty}^{\infty} \delta(t - n\Delta T)$$

Transformada de Fourier Contínua 1D

$$F\{f(t)\} = F(\mu) = \int_{-\infty}^{\infty} f(t) e^{-j2\pi\mu t} dt$$

$$f(t) = \int_{-\infty}^{\infty} F(\mu) e^{j2\pi\mu t} d\mu$$

Propriedade de Simetria

$$f(t) \rightarrow F(\mu)$$

$$F(t) \rightarrow f(-\mu)$$

Transformada de Fourier Contínua 1D

$$F\{f(t)\} = F(\mu) = \int_{-\infty}^{\infty} f(t) e^{-j2\pi\mu t} dt$$

$$f(t) = \int_{-\infty}^{\infty} F(\mu) e^{j2\pi\mu t} d\mu$$

Propriedade de Simetria

$$f(t) \rightarrow F(\mu)$$

$$F(t) \rightarrow f(-\mu)$$

Sabendo que:

$$\delta(t - t_0) \rightarrow e^{-j2\pi t_0 t}$$

temos

$$e^{-j2\pi t_0 t} \rightarrow \delta(-\mu - t_0)$$

fazendo $-t_0 = a$

$$e^{j2\pi a t} \rightarrow \delta(-\mu + a) = \delta(\mu - a)$$

Exemplo 4: Já que $s_{\Delta T}(t) = \sum_{n=-\infty}^{\infty} \delta(t - n\Delta T)$ é um sinal periódico, podemos expressar o trem de impulsos utilizando a série de Fourier

$$s_{\Delta T} = \sum_{n=-\infty}^{\infty} c_n e^{j\frac{2\pi n}{\Delta T}t}$$

onde

$$\begin{aligned} c_n &= \frac{1}{\Delta T}\int_{-\Delta T/2}^{\Delta T/2} s_{\Delta T}(t) e^{-j\frac{\pi n t}{\Delta T}} dt \\ &= \frac{1}{\Delta T}\int_{-\Delta T/2}^{\Delta T/2} \delta(t) e^{-j\frac{\pi n t}{\Delta T}} dt \\ &= \frac{1}{\Delta T} e^{O} = \frac{1}{\Delta T} \end{aligned}$$

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$$s_{\Delta T} = \frac{1}{\Delta T}\sum_{n=-\infty}^{\infty} e^{j\frac{2\pi n}{\Delta T}t}$$

Exemplo 4:

$$F\left\{s_{\Delta T}\right\} = F\left\{\frac{1}{\Delta T}\sum_{n=-\infty}^{\infty} e^{j\frac{2\pi n}{\Delta T}t}\right\}$$

$$F\left\{e^{j\frac{2\pi n}{\Delta T}t}\right\} = \delta\left(\mu - \frac{n}{\Delta T}\right)$$

Exemplo 4:

$$F\{s_{\Delta T}\} = F\left\{\frac{1}{\Delta T}\sum_{n=-\infty}^{\infty} e^{j\frac{2\pi n}{\Delta T}t}\right\}$$

$$F\left\{e^{j\frac{2\pi n}{\Delta T}t}\right\} = \delta\left(\mu - \frac{n}{\Delta T}\right)$$

$$F\{s_{\Delta T}\} = S(\mu) = F\left\{\frac{1}{\Delta T}\sum_{n=-\infty}^{\infty} e^{j\frac{2\pi n}{\Delta T}t}\right\} = \frac{1}{\Delta T}F\left\{\sum_{n=-\infty}^{\infty} e^{j\frac{2\pi n}{\Delta T}t}\right\}$$
$$= \frac{1}{\Delta T}\sum_{n=-\infty}^{\infty}\delta\left(\mu - \frac{n}{\Delta T}\right)$$

Transformada de Fourier Contínua 1D

Exemplo 4:

$$F\{s_{\Delta T}\} = F\left\{\frac{1}{\Delta T}\sum_{n=-\infty}^{\infty} e^{j\frac{2\pi n}{\Delta T}t}\right\}$$

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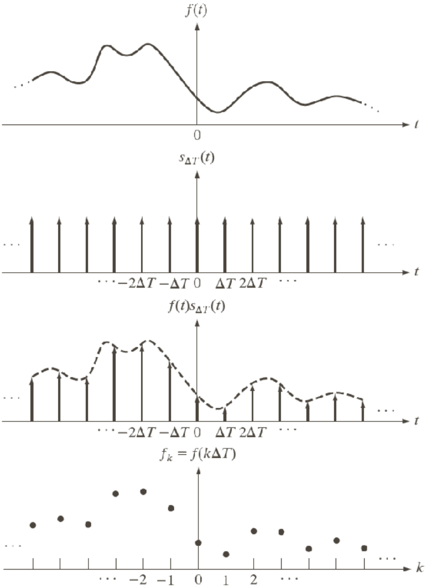
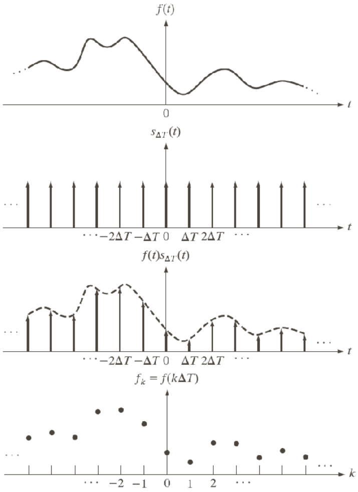
$$F\{s_{\Delta T}\} = S(\mu) = F\left\{\frac{1}{\Delta T}\sum_{n=-\infty}^{\infty} e^{j\frac{2\pi n}{\Delta T}t}\right\} = \frac{1}{\Delta T}F\left\{\sum_{n=-\infty}^{\infty} e^{j\frac{2\pi n}{\Delta T}t}\right\}$$
$$= \frac{1}{\Delta T}\sum_{n=-\infty}^{\infty}\delta\left(\mu - \frac{n}{\Delta T}\right)$$

Outro trem de impulsos!

Amostragem



f(t)
0
t
s_ΔT(t)
t
···−2ΔT −ΔT 0 ΔT 2ΔT ···
f(t)s_ΔT(t)
t
···−2ΔT −ΔT 0 ΔT 2ΔT ···
f_k = f(kΔT)
k
··· −2 −1 0 1 2 ···



Sinal amostrado:

$$\tilde{f}(t) = f(t) \cdot s_{\Delta T}(t) = \sum_{n=-\infty}^{\infty} f(t)\delta(t - n\Delta T)$$

$$f_k = \int_{-\infty}^{\infty} f(t)\delta(t - k\Delta T)dt = f(k\Delta T)$$

Tomando a Transformada de Fourier do sinal amostrado

$$\tilde{f}(t) = f(t) \cdot s_{\Delta T}(t)$$

$$f_k = \int_{-\infty}^{\infty} f(t)\delta(t - k\Delta T)dt$$

$$F\left\{\tilde{f}(t)\right\} = \tilde{F}(\mu) = F\{f(t) \cdot s_{\Delta T}(t)\}$$
$$= F(\mu) * S(\mu)$$

Tomando a Transformada de Fourier do sinal amostrado

$$\tilde{f}(t) = f(t) \cdot s_{\Delta T}(t)$$

$$f_k = \int_{-\infty}^{\infty} f(t)\delta(t - k\Delta T)dt$$

$$F\left\{\tilde{f}(t)\right\} = \tilde{F}(\mu) = F\left\{f(t) \cdot s_{\Delta T}(t)\right\}$$
$$= F(\mu) * S(\mu)$$

Mais uma propriedade: TF da Convolução

$$f(t) * h(t) = \int_{-\infty}^{\infty} f(\tau) h(t-\tau) d\tau$$

$$F\{f(t) * h(t)\} = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} f(\tau) h(t-\tau) d\tau \right] e^{-j2\pi\mu t} dt$$

$$= \int_{-\infty}^{\infty} f(\tau) \left[\int_{-\infty}^{\infty} h(t-\tau) e^{-j2\pi\mu t} dt \right] d\tau$$

Amostragem

$$f(t) * h(t) = \int_{-\infty}^{\infty} f(\tau)h(t-\tau)d\tau$$

$$F\{f(t) * h(t)\} = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} f(\tau)h(t-\tau)d\tau\right] e^{-j2\pi\mu t}dt$$

$$= \int_{-\infty}^{\infty} f(\tau) \left[\int_{-\infty}^{\infty} h(t-\tau)e^{-j2\pi\mu t}dt\right] d\tau$$

$$F\{h(t)\} = H(\mu)$$

$$F\{h(t-t_0)\} = \int_{-\infty}^{\infty} h(t-\tau)e^{-j2\pi\mu t}dt = \int_{-\infty}^{\infty} h(t_1)e^{-j2\pi\mu(t_1+\tau)}dt_1$$

$$= e^{-j2\pi\mu\tau} \int_{-\infty}^{\infty} h(t_1)e^{-j2\pi\mu t_1)}dt_1$$

$$= H(\mu)e^{-j2\pi\mu\tau}$$

Amostragem

$$F\{f(t)*h(t)\} = \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} f(\tau)h(t-\tau)d\tau\right]e^{-j2\pi\mu t}dt$$
$$= \int_{-\infty}^{\infty} f(\tau)\left[\int_{-\infty}^{\infty} h(t-\tau)e^{-j2\pi ut}dt\right]d\tau$$

$$F\{h(t-t_0)\} = H(\mu)e^{-j2\pi\mu\tau}$$

$$F\{f(t)*h(t)\} = \int_{-\infty}^{\infty} f(\tau)\left[H(\mu)e^{-j2\pi\mu\tau}\right]d\tau$$
$$= H(\mu)\int_{-\infty}^{\infty} f(\tau)e^{-j2\pi\mu\tau}d\tau = H(\mu)F(\mu)$$

Propriedades da TF

Convolução

$$f(t) * h(t) \rightarrow F(\mu)H(\mu)$$

Modulação

$$f(t) \cdot h(t) \rightarrow F(\mu) * H(\mu)$$

Amostragem

$$\tilde{F}(\mu) = F\left\{\tilde{f}(t)\right\} = \tilde{F}(\mu) = F\left\{f(t) \cdot s_{\Delta T}(t)\right\}$$
$$= F(\mu) * F\left\{s_{\Delta T}(t)\right\} = F(\mu) * S(\mu)$$

Amostragem

$$\tilde{F}(\mu) = F\left\{\tilde{f}(t)\right\} = \tilde{F}(\mu) = F\{f(t) \cdot s_{\Delta T}(t)\}$$
$$= F(\mu) * F\{s_{\Delta T}(t)\} = F(\mu) * S(\mu)$$

$$S(\mu) = \frac{1}{\Delta T} \sum_{n=-\infty}^{\infty} \delta\left(\mu - \frac{n}{\Delta T}\right)$$

Amostragem

$$\tilde{F}(\mu) = F\left\{\tilde{f}(t)\right\} = \tilde{F}(\mu) = F\left\{f(t) \cdot s_{\Delta T}(t)\right\}$$
$$= F(\mu) * F\left\{s_{\Delta T}(t)\right\} = F(\mu) * S(\mu)$$

$$S(\mu) = \frac{1}{\Delta T} \sum_{n=-\infty}^{\infty} \delta\left(\mu - \frac{n}{\Delta T}\right)$$

$$\tilde{F}(\mu) = F(\mu) * S(\mu) = \int_{-\infty}^{\infty} F(\tau) S(\mu - \tau) d\tau$$
$$= \frac{1}{\Delta T} \int_{-\infty}^{\infty} F(\tau) \sum_{n=-\infty}^{\infty} \delta\left(\mu - \tau - \frac{n}{\Delta T}\right) d\tau$$

Amostragem

$$\tilde{F}(\mu) = F(\mu) * S(\mu) = \int_{-\infty}^{\infty} F(\tau) S(\mu - \tau) d\tau$$

$$= \frac{1}{\Delta T} \int_{-\infty}^{\infty} F(\tau) \sum_{n=-\infty}^{\infty} \delta\left(\mu - \tau - \frac{n}{\Delta T}\right) d\tau$$

$$= \frac{1}{\Delta T} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} F(\tau) \delta\left(\mu - \tau - \frac{n}{\Delta T}\right) d\tau$$

$$= \frac{1}{\Delta T} \sum_{n=-\infty}^{\infty} F\left(\mu - \frac{n}{\Delta T}\right)$$

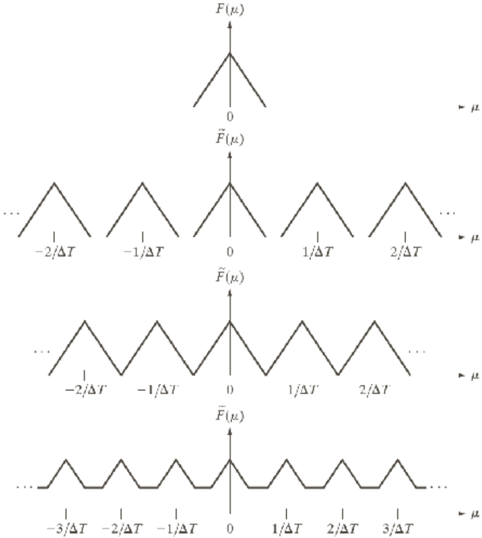
Amostragem

$$\begin{aligned}\tilde{F}(\mu) = F(\mu) * S(\mu) &= \int_{-\infty}^{\infty} F(\tau) S(\mu - \tau) d\tau \\ &= \frac{1}{\Delta T} \int_{-\infty}^{\infty} F(\tau) \sum_{n=-\infty}^{\infty} \delta\left(\mu - \tau - \frac{n}{\Delta T}\right) d\tau \\ &= \frac{1}{\Delta T} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} F(\tau) \delta\left(\mu - \tau - \frac{n}{\Delta T}\right) d\tau \\ &= \frac{1}{\Delta T} \sum_{n=-\infty}^{\infty} F\left(\mu - \frac{n}{\Delta T}\right)\end{aligned}$$

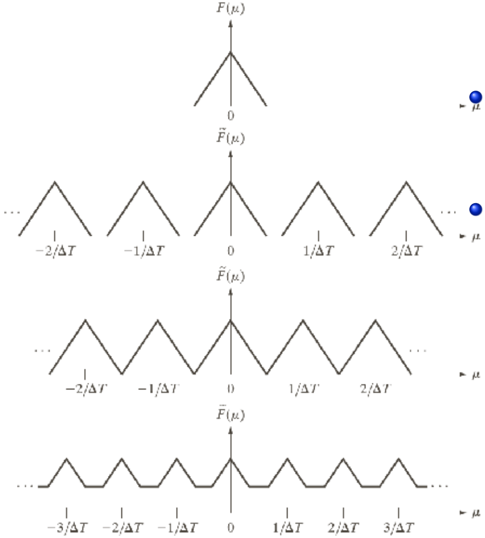
A TF $\tilde{F}(\mu)$ do sinal amostrado $\tilde{f}(t)$:

- É uma sequência (periódica e infinita) de cópias de $F(\mu)$ deslocadas;
- A separação entre as cópias é $\frac{1}{\Delta T}$;
- É uma função contínua ($F(\mu)$ é contínua);

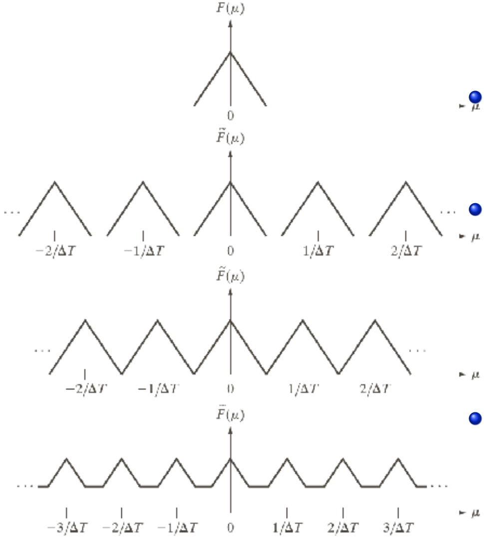
Amostragem



$F(\mu)$
0
μ
$\tilde{F}(\mu)$
...
$-2/\Delta T$
$-1/\Delta T$
0
$1/\Delta T$
$2/\Delta T$
...
μ
$\tilde{F}(\mu)$
...
$-2/\Delta T$
$-1/\Delta T$
0
$1/\Delta T$
$2/\Delta T$
...
μ
$\tilde{F}(\mu)$
...
$-3/\Delta T$
$-2/\Delta T$
$-1/\Delta T$
0
$1/\Delta T$
$2/\Delta T$
$3/\Delta T$
...
μ



- $\frac{1}{\Delta T}$ é a taxa de amostragem usada para gerar o sinal discreto;
- Na figura, temos o espectro para valores de $\frac{1}{\Delta T}$:
  - grande "suficiente";
  - "suficiente";
  - menor do que o mínimo necessário.



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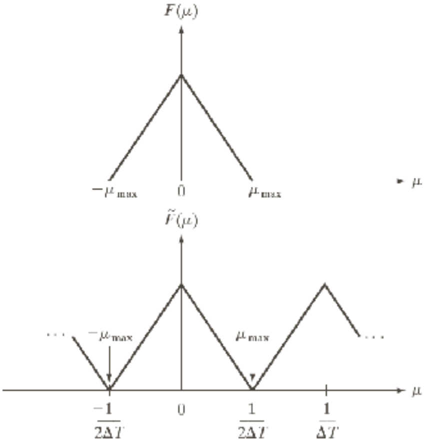
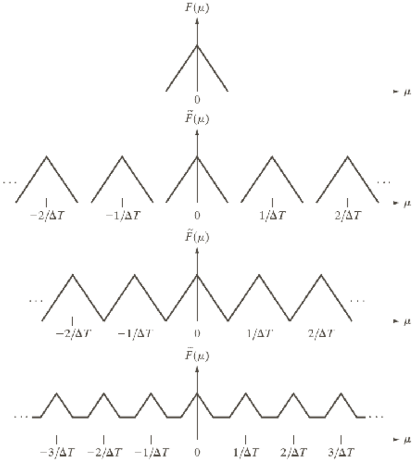
- Na figura, temos o espectro para valores de $\frac{1}{\Delta T}$:
  - grande "suficiente";
  - "suficiente";
  - menor do que o mínimo necessário.
- Qual é o valor mínimo?

Assumindo que um sinal $f(t)$ tenha transformada de Fourier com valores nulos fora do intervalo $[-\mu_{max}, \mu_{max}]$, ou seja, o sinal tem banda limitada.

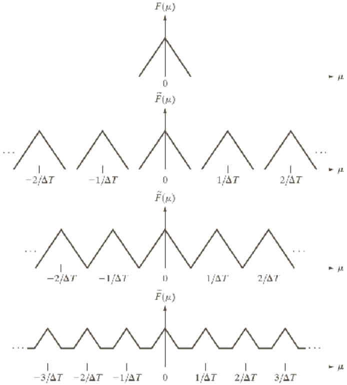


Que condição devemos satisfazer para que o sinal seja recuperado sem maiores problemas? (Sem sobreposição dos sinais?)



$$\frac{1}{\Delta T} > 2\mu_{max} = f_s$$

Taxa de Nyquist – Teorema de Amostragem



Teorema da Amostragem

Como recuperar o sinal?

Como recuperar o sinal? Considere um sinal $h(t)$, cuja transformada de Fourier é dada por

$$H(\mu) = \begin{cases} \Delta T & \text{se } -u_{max} \leq \mu \leq \mu_{max} \\ 0 & \text{caso contrário} \end{cases}$$

Quando multiplicamos a sequência periódica por esta função

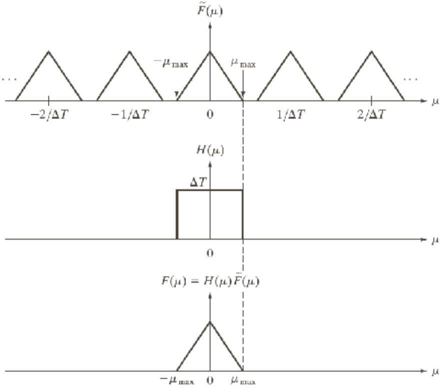
$$F(\mu) = H(\mu)\tilde{F}(\mu)$$

Logo, obtemos $f(t)$ tirando a T.F. inversa de $F(\mu)$

$$f(t) = \int_{-\infty}^{\infty} F(\mu)e^{j2\pi\mu t}d\mu$$

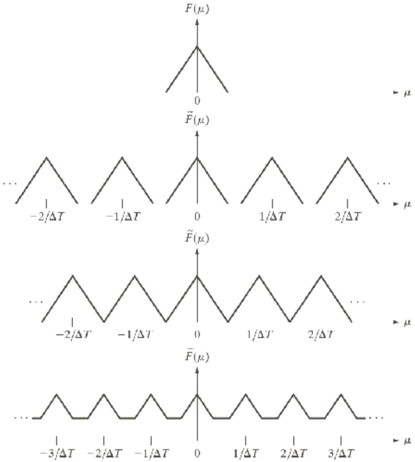
Filtro passa-baixas

$$H(\mu) = \begin{cases} \Delta T & \text{se } -u_{max} \leq \mu \leq \mu_{max} \\ 0 & \text{caso contrário} \end{cases}$$

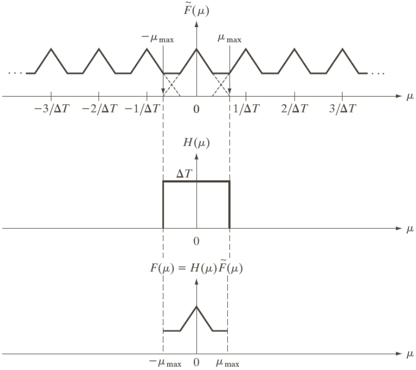


O que acontece quando não respeitamos a taxa de Nyquist?

O que acontece quando não respeitamos a taxa de Nyquist? ALIAS!



O que acontece quando não respeitamos a taxa de Nyquist? ALIAS!



- Alias não pode ser evitado ... Mesmo que o sinal seja limitado em banda, ao limitar o sinal no tempo introduzimos frequências infinitas;

$$h_p(t) = \begin{cases} 1 & \text{se } 0 \leq t \leq T \\ 0 & \text{caso contrário} \end{cases}$$

- A convolução de $h_p(t)$ e $f(t)$ vai gerar um sinal com infinitas frequências;

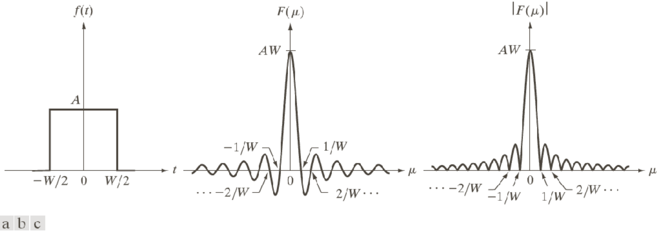


FIGURE 4.4 (a) A simple function; (b) its Fourier transform; and (c) the spectrum. All functions extend to infinity in both directions.

Teorema de Amostragem

$$
\begin{aligned}
f(t) &= \int_{-\infty}^{\infty} F(\mu) e^{j2\pi\mu t} d\mu \\
&= \int_{-\infty}^{\infty} \left(H(\mu)\tilde{F}(\mu) \right) e^{j2\pi\mu t} d\mu \\
&= h(t) * \tilde{f}(t)
\end{aligned}
$$

Teorema de Amostragem

$$
\begin{aligned}
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&= h(t) * \tilde{f}(t)
\end{aligned}
$$

como

$$\tilde{f}(t) = f(t) s_{\Delta T}(t) = \sum_{n=-\infty}^{\infty} f(t)\delta(t - n\Delta T)$$

Teorema de Amostragem

$$
\begin{aligned}
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&= h(t) * \tilde{f}(t)
\end{aligned}
$$

como

$$
\tilde{f}(t) = f(t) s_{\Delta T}(t) = \sum_{n=-\infty}^{\infty} f(t)\delta(t - n\Delta T)
$$

$$
f(t) = \sum_{-\infty}^{\infty} f(n\Delta T) \text{sinc}\left[(t - n\Delta T)/n\Delta T\right]
$$

- A TF de um sinal de um sinal amostrado e limitado em banda, se extende no tempo de $-\infty$ a ∞ e é uma função *contínua* e *periódica*;

Transformada Discreta de Fourier

- A TF de um sinal de um sinal amostrado e limitado em banda, se extende no tempo de $-\infty$ a ∞ e é uma função *contínua* e *periódica*;
- Na prática, devemos ter um número finito de amostras;
- E uma transformada adequada.

Transformada Discreta de Fourier

- A TF de um sinal de um sinal amostrado e limitado em banda, se extende no tempo de $-\infty$ a ∞ e é uma função *contínua* e *periódica*;
- Na prática, devemos ter um número finito de amostras;
- E uma transformada adequada.

$$\begin{aligned}
\tilde{F}(\mu) &= \int_{-\infty}^{\infty} \tilde{f}(t) e^{-j2\pi\mu t} d\mu \\
&= \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} f(t)\delta(t - n\Delta T) e^{-j2\pi\mu t} d\mu \\
&= \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} f(t)\delta(t - n\Delta T) e^{-j2\pi\mu t} d\mu \\
&= \sum_{n=-\infty}^{\infty} f_n e^{-j2\pi\mu n\Delta T}
\end{aligned}$$

- Como $\tilde{F}(\mu)$ é contínuo e periódico com período $1/\Delta T$
- Logo, para caracterização do sinal precisamos apenas de 1 período amostrado de $\tilde{F}(\mu)$.

- Como $\tilde{F}(\mu)$ é contínuo e periódico com período $1/\Delta T$
- Logo, para caracterização do sinal precisamos apenas de 1 período amostrado de $\tilde{F}(\mu)$.
- Suponha que em um período $(0 \leq \mu \leq 1/\Delta T)$ de $\tilde{F}(\mu)$ queremos coletar M amostras:

$$\mu = \frac{m}{M\Delta T}, \qquad m = 0, 1, 2, \ldots, M-1.$$

- Como $\tilde{F}(\mu)$ é contínuo e periódico com período $1/\Delta T$
- Logo, para caracterização do sinal precisamos apenas de 1 período amostrado de $\tilde{F}(\mu)$.
- Suponha que em um período $(0 \leq \mu \leq 1/\Delta T)$ de $\tilde{F}(\mu)$ queremos coletar M amostras:

$$\mu = \frac{m}{M\Delta T}, \qquad m = 0, 1, 2, \ldots, M-1.$$

$$\tilde{F}(\mu) = \sum_{n=-\infty}^{\infty} f_n e^{-j2\pi\mu n\Delta T}$$

$$F_m = \sum_{n=0}^{M-1} f_n e^{-j2\pi mn/M} \quad m = 0, 1, 2, \ldots, M-1.$$

Esta é expressão da TF discreta que estamos procurando.

Transformada de Discreta de Fourier

Transformada Discreta de Fourier (TDF) - Direta

$$F(u) = \sum_{x=0}^{M-1} f(x) e^{-j2\pi ux/M} \quad m = 0, 1, 2, \dots, M-1.$$

Transformada Discreta de Fourier (TDF) - Inversa

$$f(x) = \frac{1}{M} \sum_{u=0}^{M-1} F(u) e^{j2\pi ux/M} \quad x = 0, 1, 2, \dots, M-1.$$

Transformada de Discreta de Fourier

$$F(u) = F(u + kM)$$

$$f(x) = f(x + kM)$$

$$F(u) = F(u + kM)$$

$$f(x) = f(x + kM)$$

E a convolução? Convolução Circular

$$f(x) * h(x) = \sum_{m=0}^{M-1} f(m)h(x - m)$$

$$F(u) = F(u + kM)$$

$$f(x) = f(x + kM)$$

E a convolução? Convolução Circular

$$f(x) * h(x) = \sum_{m=0}^{M-1} f(m)h(x - m)$$

Relação entre a amostragem e os intervalos de frequência

$$T = M\Delta T$$

$$\Delta u = 1/(M\Delta T) = 1/T$$

$$\Omega = M\Delta u = 1/\Delta T$$

$$X[k] = \begin{cases} \sum_{n=0}^{N-1} x[n] W_N^{kn}, & 0 \leq k \leq N-1 \\ 0, & \text{c.c.} \end{cases},$$

$$x[n] = \begin{cases} 1/N \sum_{k=0}^{N-1} X[k] W_N^{-kn}, & 0 \leq n \leq N-1 \\ 0, & \text{c.c.} \end{cases}.$$

A partir destas relações que definimos a Transformada Discreta de Fourier (TDF) de N amostras.

Análise:

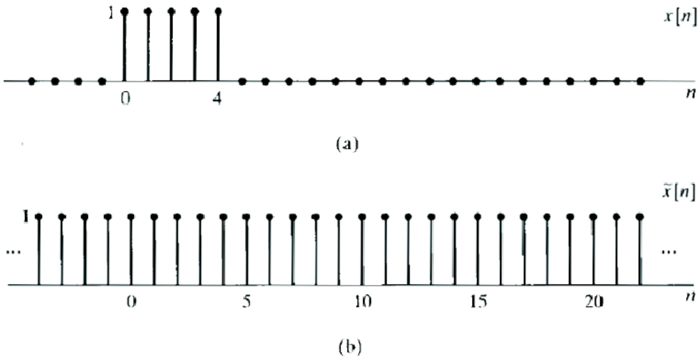
$$\boxed{X[k] = \sum_{n=0}^{N-1} x[n] W_N^{kn}} \quad 0 \leq k \leq N-1, \tag{18}$$

$$W_N = e^{-j2\pi n/N}.$$

Síntese:

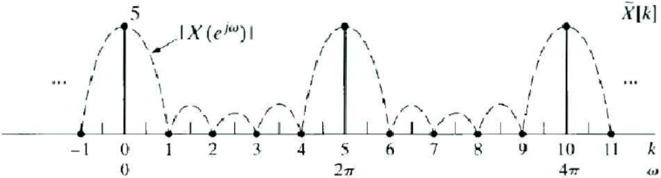
$$\boxed{x[n] = 1/N \sum_{k=0}^{N-1} X[k] W_N^{-kn}}, \quad 0 \leq n \leq N-1. \tag{19}$$

Cálculo da TDF de um degrau de comprimento 5:

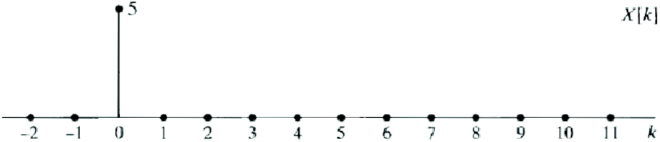


Cálculo da TDF de um degrau de comprimento 5:

$$\tilde{X}[k] = \sum_{n=0}^{4} 1e^{-j(2\pi/5)\cdot kn} = 5\delta[k - r \cdot 5]$$



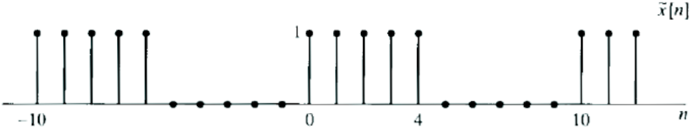
(c)



Exemplo

Cálculo da TDF de um degrau de comprimento 10:

(a)

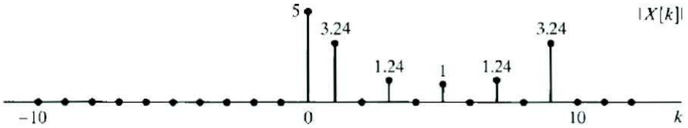


(b)

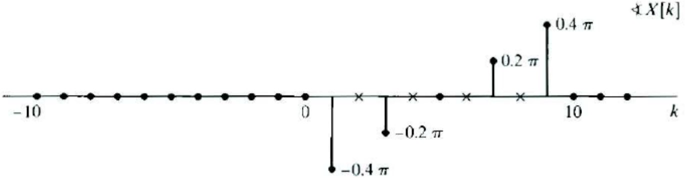
Cálculo da TDF de um degrau de comprimento 10:

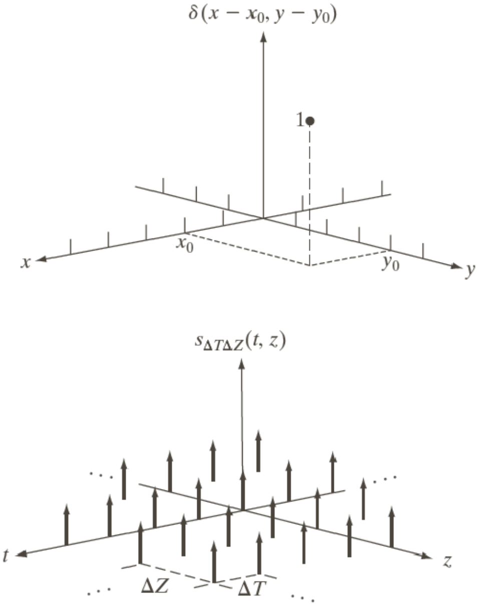
$$\tilde{X}[k] = \sum_{n=0}^{4} 1e^{-j(2\pi/10)\cdot kn} = \frac{1 - e^{-j(2\pi)k}}{1 - e^{-j(2\pi/5)k}} = e^{-j(4\pi/10)k}\,\frac{\sin(\pi k/2)}{\sin(\pi k/10)}$$

(b)

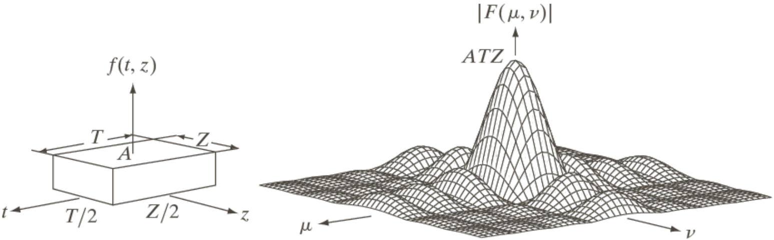


(c)




δ(x − x0, y − y0)
1
x
x0
y0
y
sΔTΔZ(t, z)
t
z
ΔZ
ΔT

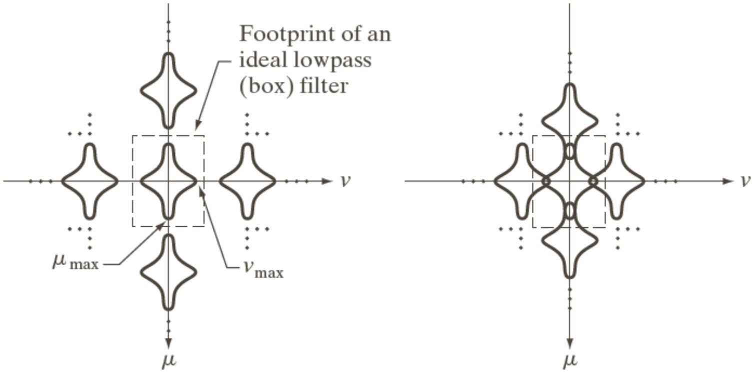
Sinais 2-D



a b

FIGURE 4.13 (a) A 2-D function, and (b) a section of its spectrum (not to scale). The block is longer along the t-axis, so the spectrum is more "contracted" along the μ-axis. Compare with Fig. 4.4.

Sinais 2-D



Transformada de Fourier Discreta 2D

$$F(u,v) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{-j(2\pi)\cdot\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

Transformada de Fourier Discreta 2D

$$F(u,v) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{-j(2\pi)\cdot\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

$$f(x,y) = \sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v)e^{j(2\pi)\cdot\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

$$F(u,v) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-j(2\pi)\cdot\left(\frac{ux}{M} + \frac{vy}{N}\right)}$$

$$f(x,y) = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u,v) e^{j(2\pi)\cdot\left(\frac{ux}{M} + \frac{vy}{N}\right)}$$

Magnitude:

$$|F(u,v)| = \left[R^2(u,v) + I^2(u,v)\right]^{1/2}$$

Fase:

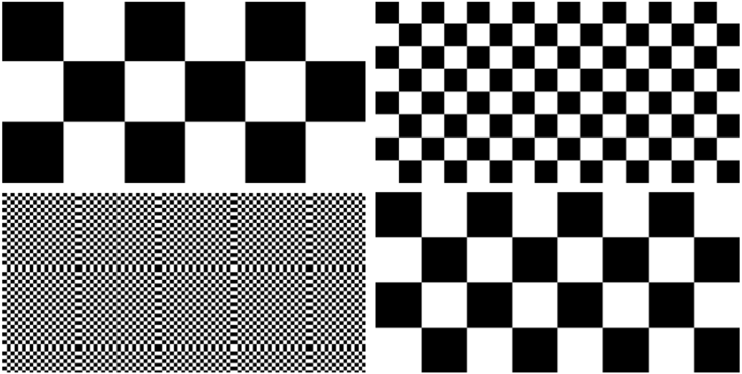
$$|\phi(u,v)| = \arctan\left[\frac{I(u,v)}{R(u,v)}\right]$$

Fourier transform	$F(u,v) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{-j2\pi(ux/M+vy/N)}$
Inverse Fourier transform	$f(x,y) = \sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v)e^{j2\pi(ux/M+vy/N)}$
Polar representation	$F(u,v) = \lvert F(u,v)\rvert e^{-j\phi(u,v)}$
Spectrum	$\lvert F(u,v)\rvert = \left[R^2(u,v) + I^2(u,v)\right]^{1/2}$, $R = \text{Real}(F)$ and $I = \text{Imag}(F)$
Phase angle	$\phi(u,v) = \tan^{-1}\left[\frac{I(u,v)}{R(u,v)}\right]$
Power spectrum	$P(u,v) = \lvert F(u,v)\rvert^2$
Average value	$\bar{f}(x,y) = F(0,0) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)$
Translation	$f(x,y)e^{j2\pi(u_0x/M+v_0y/N)} \Leftrightarrow F(u-u_0, v-v_0)$ $f(x-x_0, y-y_0) \Leftrightarrow F(u,v)e^{-j2\pi(ux_0/M+vy_0/N)}$ When $x_0 = u_0 = M/2$ and $y_0 = v_0 = N/2$, then $f(x,y)(-1)^{x+y} \Leftrightarrow F(u-M/2, v-N/2)$ $f(x-M/2, y-N/2) \Leftrightarrow F(u,v)(-1)^{u+v}$

Propriedades

Conjugate symmetry	$F(u,v) = F^*(-u,-v)$ $\lvert F(u,v)\rvert = \lvert F(-u,-v)\rvert$
Differentiation	$\dfrac{\partial^n f(x,y)}{\partial x^n} \Leftrightarrow (ju)^n F(u,v)$ $(-jx)^n f(x,y) \Leftrightarrow \dfrac{\partial^n F(u,v)}{\partial u^n}$
Laplacian	$\nabla^2 f(x,y) \Leftrightarrow -(u^2+v^2)F(u,v)$
Distributivity	$\Im[f_1(x,y) + f_2(x,y)] = \Im[f_1(x,y)] + \Im[f_2(x,y)]$ $\Im[f_1(x,y)\cdot f_2(x,y)] \neq \Im[f_1(x,y)]\cdot\Im[f_2(x,y)]$
Scaling	$af(x,y) \Leftrightarrow aF(u,v),\ f(ax,by) \Leftrightarrow \dfrac{1}{\lvert ab\rvert}F(u/a, v/b)$
Rotation	$x = r\cos\theta \quad y = r\sin\theta \quad u = \omega\cos\varphi \quad v = \omega\sin\varphi$ $f(r,\theta+\theta_0) \Leftrightarrow F(\omega,\varphi+\theta_0)$
Periodicity	$F(u,v) = F(u+M,v) = F(u,v+N) = F(u+M,v+N)$ $f(x,y) = f(x+M,y) = f(x,y+N) = f(x+M,y+N)$
Separability	See Eqs. (4.6-14) and (4.6-15). Separability implies that we can compute the 2-D transform of an image by first computing 1-D transforms along each row of the image, and then computing a 1-D transform along each column of this intermediate result. The reverse, columns and then rows, yields the same result.

2D Aliasing

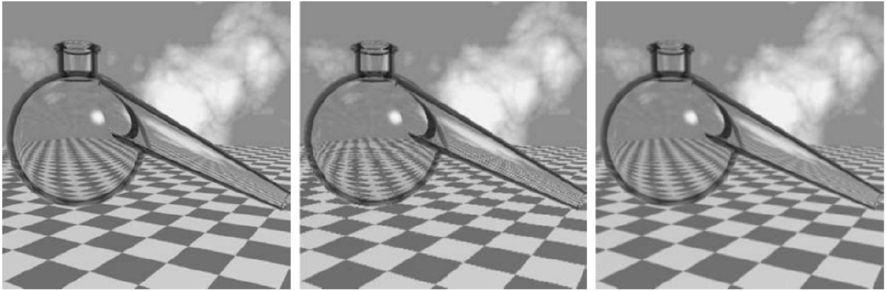


a b
c d

FIGURE 4.16 Aliasing in images. In (a) and (b), the lengths of the sides of the squares are 16 and 6 pixels, respectively, and aliasing is visually negligible. In (c) and (d), the sides of the squares are 0.9174 and 0.4798 pixels, respectively, and the results show significant aliasing. Note that (d) masquerades as a "normal" image.

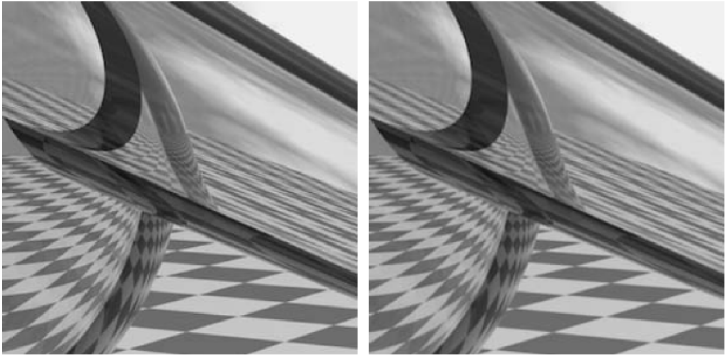
a b c

FIGURE 4.17 Illustration of aliasing on resampled images. (a) A digital image with negligible visual aliasing. (b) Result of resizing the image to 50% of its original size by pixel deletion. Aliasing is clearly visible. (c) Result of blurring the image in (a) with a 3×3 averaging filter prior to resizing. The image is slightly more blurred than (b), but aliasing is not longer objectionable. (Original image courtesy of the Signal Compression Laboratory, University of California, Santa Barbara.)



a b c

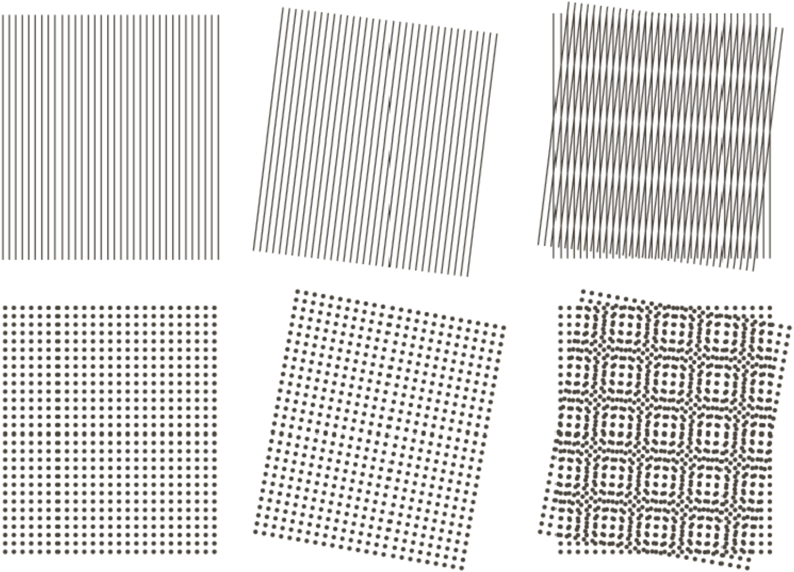
FIGURE 4.18 Illustration of jaggies. (a) A 1024 × 1024 digital image of a computer-generated scene with negligible visible aliasing. (b) Result of reducing (a) to 25% of its original size using bilinear interpolation. (c) Result of blurring the image in (a) with a 5 × 5 averaging filter prior to resizing it to 25% using bilinear interpolation. (Original image courtesy of D. P. Mitchell, Mental Landscape, LLC.)



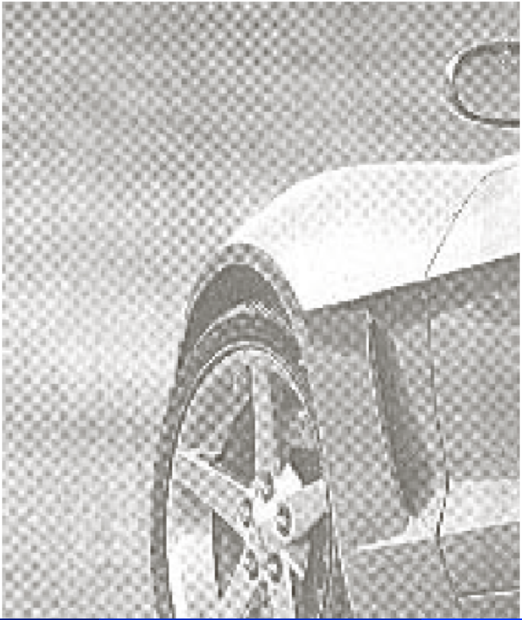
a b

FIGURE 4.19 Image zooming. (a) A 1024 × 1024 digital image generated by pixel replication from a 256 × 256 image extracted from the middle of Fig. 4.18(a). (b) Image generated using bi-linear interpolation, showing a significant reduction in jaggies.

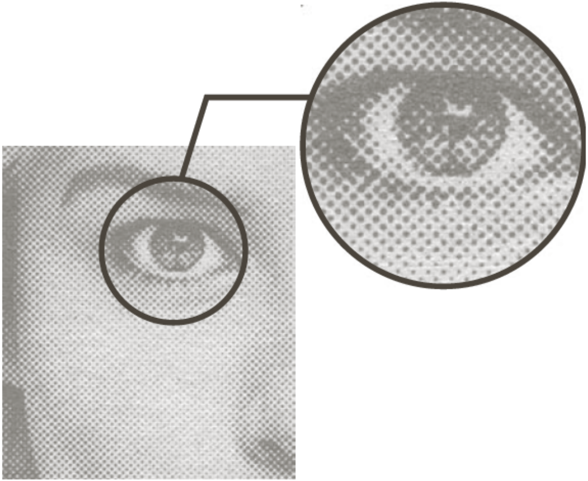
Efeito Moiré



Efeito Moiré



Efeito Moiré

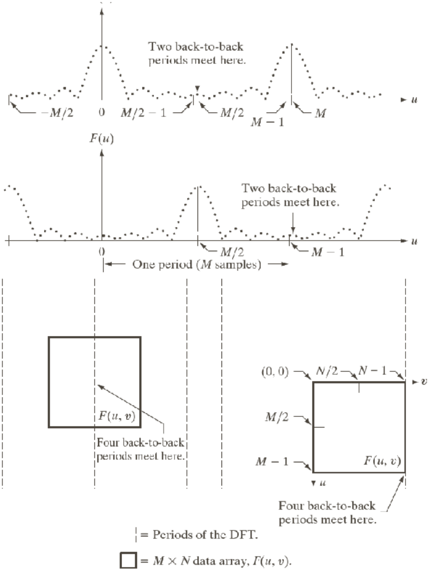


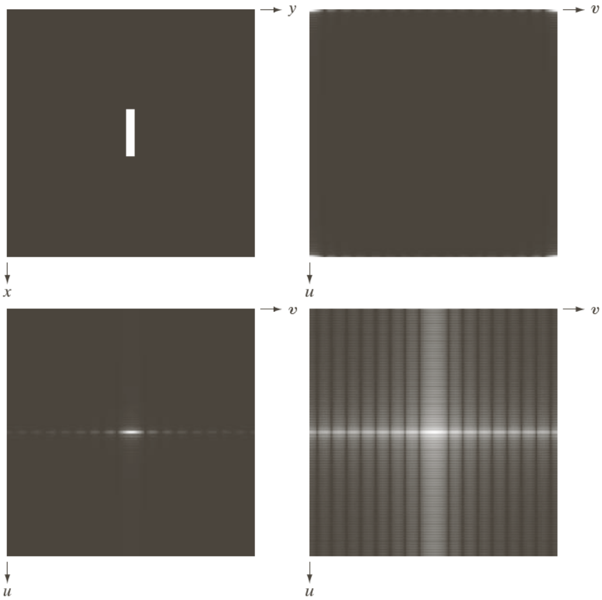
Propriedades TFD-2D

	Spatial Domain†		Frequency Domain†
1)	$f(x, y)$ real	$\Leftrightarrow$	$F^*(u, v) = F(-u, -v)$
2)	$f(x, y)$ imaginary	$\Leftrightarrow$	$F^*(-u, -v) = -F(u, v)$
3)	$f(x, y)$ real	$\Leftrightarrow$	$R(u, v)$ even; $I(u, v)$ odd
4)	$f(x, y)$ imaginary	$\Leftrightarrow$	$R(u, v)$ odd; $I(u, v)$ even
5)	$f(-x, -y)$ real	$\Leftrightarrow$	$F^*(u, v)$ complex
6)	$f(-x, -y)$ complex	$\Leftrightarrow$	$F(-u, -v)$ complex
7)	$f^*(x, y)$ complex	$\Leftrightarrow$	$F^*(-u - v)$ complex
8)	$f(x, y)$ real and even	$\Leftrightarrow$	$F(u, v)$ real and even
9)	$f(x, y)$ real and odd	$\Leftrightarrow$	$F(u, v)$ imaginary and odd
10)	$f(x, y)$ imaginary and even	$\Leftrightarrow$	$F(u, v)$ imaginary and even
11)	$f(x, y)$ imaginary and odd	$\Leftrightarrow$	$F(u, v)$ real and odd
12)	$f(x, y)$ complex and even	$\Leftrightarrow$	$F(u, v)$ complex and even
13)	$f(x, y)$ complex and odd	$\Leftrightarrow$	$F(u, v)$ complex and odd

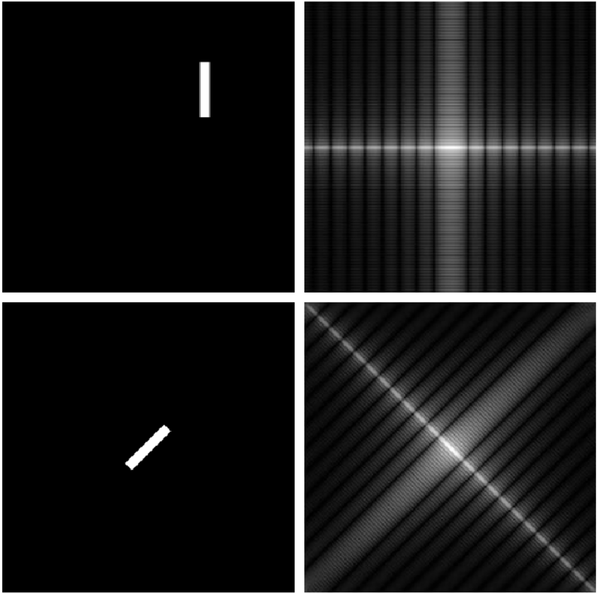
†Recall that x, y, u, and v are *discrete* (integer) variables, with x and u in the range $[0, M-1]$, and y, and v in the range $[0, N-1]$. To say that a complex function is *even* means that its real *and* imaginary parts are even, and similarly for an odd complex function.

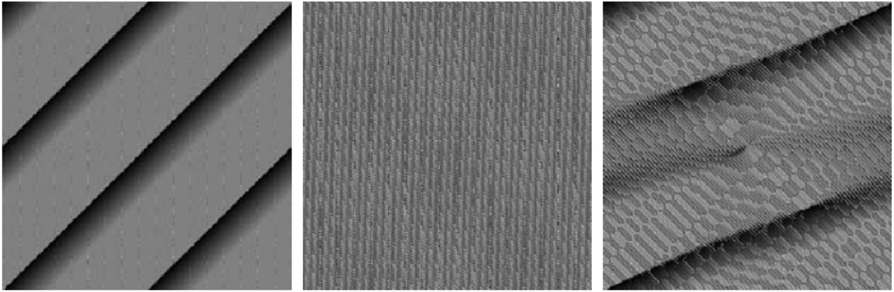
Propriedades TFD-2D




y
x
v
u
v
u
v
u

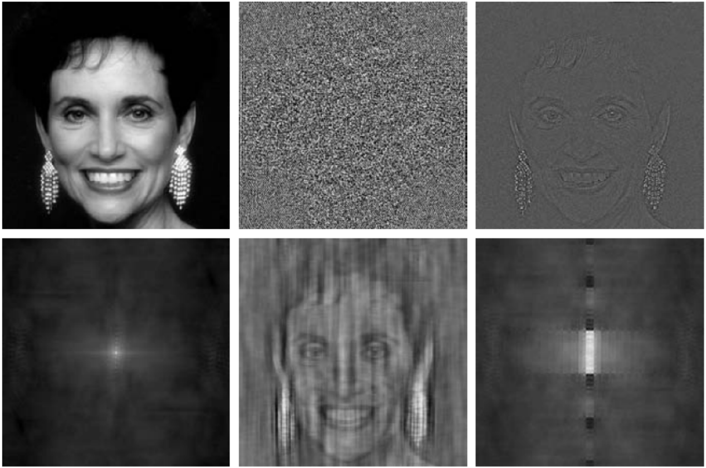
Propriedades TFD-2D





a b c

FIGURE 4.26 Phase angle array corresponding (a) to the image of the centered rectangle in Fig. 4.24(a), (b) to the translated image in Fig. 4.25(a), and (c) to the rotated image in Fig. 4.25(c).



a	b	c
d	e	f

FIGURE 4.27 (a) Woman. (b) Phase angle. (c) Woman reconstructed using only the phase angle. (d) Woman reconstructed using only the spectrum. (e) Reconstruction using the phase angle corresponding to the woman and the spectrum corresponding to the rectangle in Fig. 4.24(a). (f) Reconstruction using the phase of the rectangle and the

Name	DFT Pairs
1) Symmetry properties	See Table 4.1
2) Linearity	$af_1(x, y) + bf_2(x, y) \Leftrightarrow aF_1(u, v) + bF_2(u, v)$
3) Translation (general)	$f(x, y)e^{j2\pi(u_0x/M + v_0y/N)} \Leftrightarrow F(u - u_0, v - v_0)$ $f(x - x_0, y - y_0) \Leftrightarrow F(u, v)e^{-j2\pi(ux_0/M + vy_0/N)}$
4) Translation to center of the frequency rectangle, $(M/2, N/2)$	$f(x, y)(-1)^{x+y} \Leftrightarrow F(u - M/2, v - N/2)$ $f(x - M/2, y - N/2) \Leftrightarrow F(u, v)(-1)^{u+v}$
5) Rotation	$f(r, \theta + \theta_0) \Leftrightarrow F(\omega, \varphi + \theta_0)$ $x = r\cos\theta \quad y = r\sin\theta \quad u = \omega\cos\varphi \quad v = \omega\sin\varphi$
6) Convolution theorem†	$f(x, y) \star h(x, y) \Leftrightarrow F(u, v)H(u, v)$ $f(x, y)h(x, y) \Leftrightarrow F(u, v) \star H(u, v)$

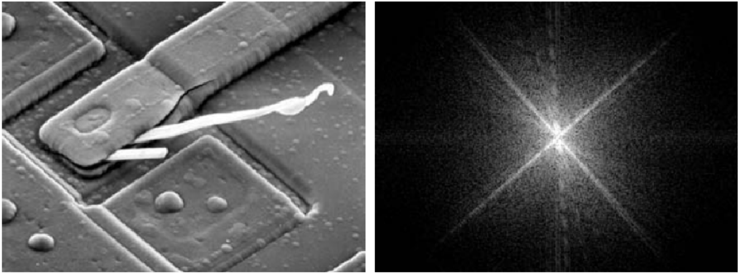
(Continued)

Propriedades TFD-2D

Name	DFT Pairs
7) Correlation theorem[†]	$f(x, y) \star h(x, y) \Leftrightarrow F^*(u, v)\,H(u, v)$ $f^*(x, y)h(x, y) \Leftrightarrow F(u, v) \star H(u, v)$
8) Discrete unit impulse	$\delta(x, y) \Leftrightarrow 1$
9) Rectangle	$\text{rect}[a, b] \Leftrightarrow ab\dfrac{\sin(\pi ua)}{(\pi ua)}\dfrac{\sin(\pi vb)}{(\pi vb)}e^{-j\pi(ua+vb)}$
10) Sine	$\sin(2\pi u_0 x + 2\pi v_0 y) \Leftrightarrow$ $j\frac{1}{2}\left[\delta(u + Mu_0, v + Nv_0) - \delta(u - Mu_0, v - Nv_0)\right]$
11) Cosine	$\cos(2\pi u_0 x + 2\pi v_0 y) \Leftrightarrow$ $\frac{1}{2}\left[\delta(u + Mu_0, v + Nv_0) + \delta(u - Mu_0, v - Nv_0)\right]$

The following Fourier transform pairs are derivable only for continuous variables, denoted as before by t and z for spatial variables and by μ and ν for frequency variables. These results can be used for DFT work by sampling the continuous forms.

12) *Differentiation* (The expressions on the right assume that $f(\pm\infty, \pm\infty) = 0$.)	$\left(\frac{\partial}{\partial t}\right)^m\left(\frac{\partial}{\partial z}\right)^n f(t, z) \Leftrightarrow (j2\pi\mu)^m(j2\pi\nu)^n F(\mu, \nu)$ $\frac{\partial^m f(t, z)}{\partial t^m} \Leftrightarrow (j2\pi\mu)^m F(\mu, \nu);\ \frac{\partial^n f(t, z)}{\partial z^n} \Leftrightarrow (j2\pi\nu)^n F(\mu, \nu)$
13) *Gaussian*	$A2\pi\sigma^2 e^{-2\pi^2\sigma^2(t^2+z^2)} \Leftrightarrow Ae^{-(\mu^2+\nu^2)/2\sigma^2}$ (A is a constant)

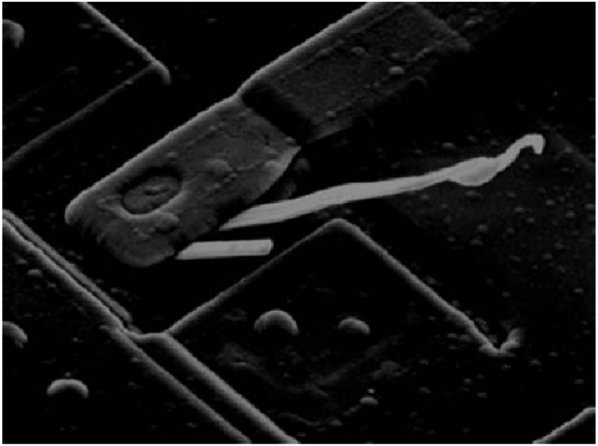


a b

FIGURE 4.29 (a) SEM image of a damaged integrated circuit. (b) Fourier spectrum of (a). (Original image courtesy of Dr. J. M. Hudak, Brockhouse Institute for Materials Research, McMaster University, Hamilton, Ontario, Canada.)

$$G(u, v) = H(u, v) \cdot F(u, v)$$

Resultado filtragem, zerando $F(M/2, N/2)$ – eliminando o nvel DC.



Passa-Baixas | Passa-Altas | Passa-Altas + DC

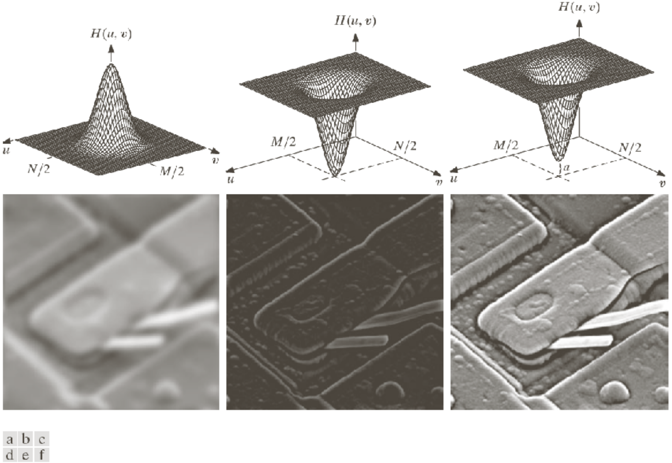
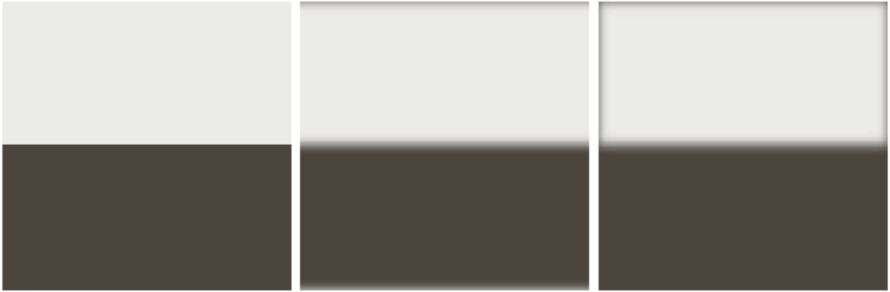


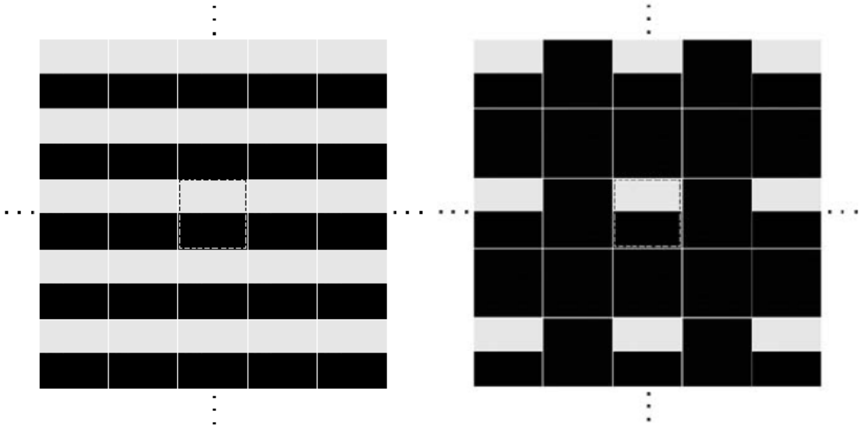
FIGURE 4.31 Top row: frequency domain filters. Bottom row: corresponding filtered images obtained using Eq. (4.7-1). We used $a = 0.85$ in (c) to obtain (f) (the height of the filter itself is 1). Compare (f) with Fig. 4.29(a).

Padding



a b c

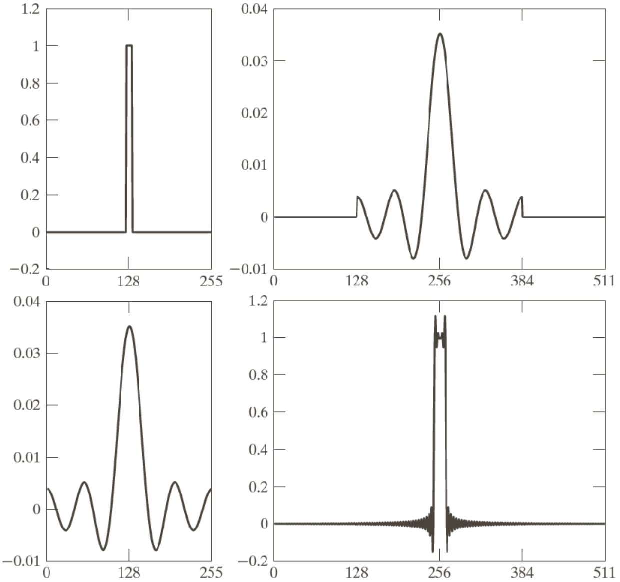
FIGURE 4.32 (a) A simple image. (b) Result of blurring with a Gaussian lowpass filter without padding. (c) Result of lowpass filtering with padding. Compare the light area of the vertical edges in (b) and (c).



a b

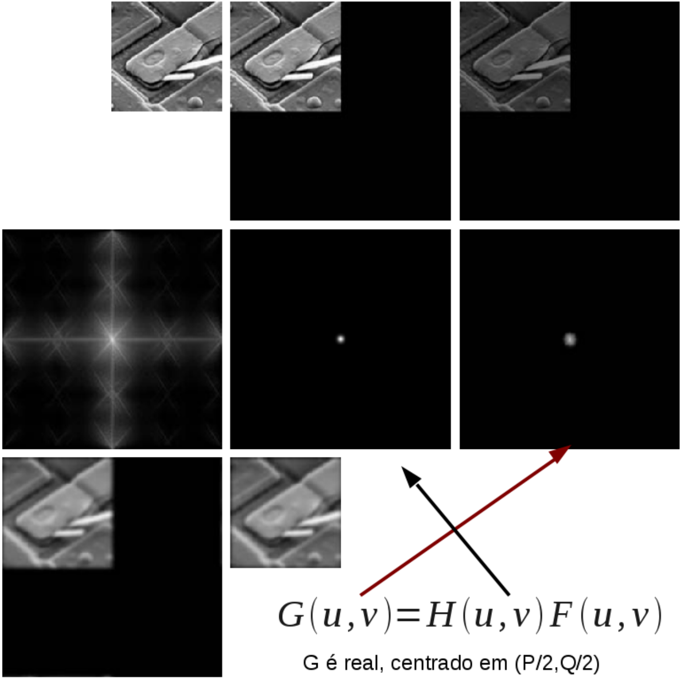
FIGURE 4.33 2-D image periodicity inherent in using the DFT. (a) Periodicity without image padding. (b) Periodicity after padding with 0s (black). The dashed areas in the center correspond to the image in Fig. 4.32(a). (The thin white lines in both images are superimposed for clarity; they are not part of the data.)

Periodicidade



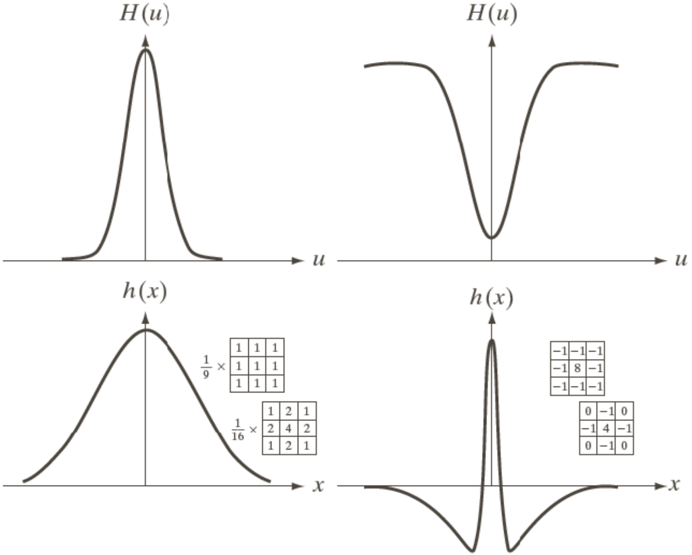
a c
b d

FIGURE 4.34 (a) Original filter specified in the (centered) frequency domain. (b) Spatial representation obtained by computing the IDFT of (a). (c) Result of padding (b) to twice its length (note the discontinuities). (d) Corresponding filter in the frequency domain obtained by computing the DFT of (c). Note the ringing caused by the discontinuities in (c). (The curves appear continuous because the points were joined to simplify visual analysis.)



a b c
d e f
g h

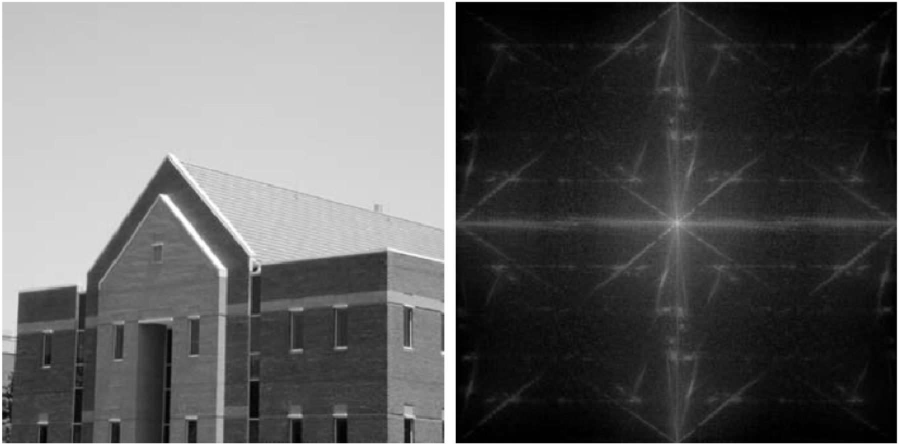
FIGURE 4.36 (a) An $M \times N$ image, f. (b) Padded image, f_p of size $P \times Q$. (c) Result of multiplying f_p by $(-1)^{x+y}$. (d) Spectrum of F_p. (e) Centered Gaussian lowpass filter, H, of size $P \times Q$. (f) Spectrum of the product HF_p. (g) g_p, the product of $(-1)^{x+y}$ and the real part of the IDFT of HF_p. (h) Final result, g, obtained by cropping the first M rows and N columns of g_p.



a c
b d

FIGURE 4.37
(a) A 1-D Gaussian lowpass filter in the frequency domain. (b) Spatial lowpass filter corresponding to (a). (c) Gaussian highpass filter in the frequency domain. (d) Spatial highpass filter corresponding to (c). The small 2-D masks shown are spatial filters we used in Chapter 3.

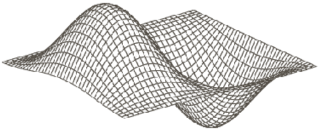
Exemplo

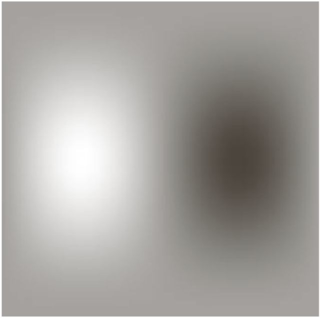


a b

FIGURE 4.38 (a) Image of a building, and (b) its spectrum.

−1	0	1
−2	0	2
−1	0	1

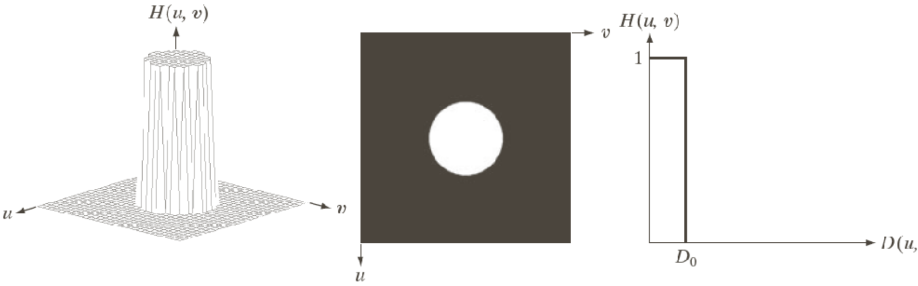




a b
c d

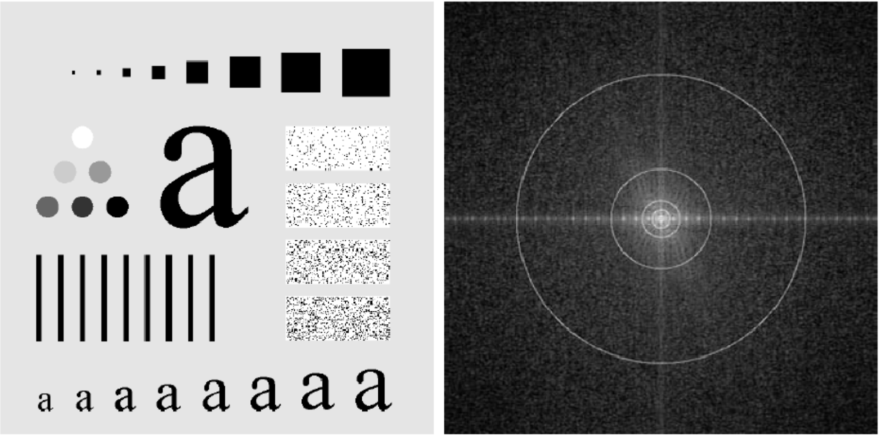
FIGURE 4.39 (a) A spatial mask and perspective plot of its corresponding frequency domain filter. (b) Filter shown as an image. (c) Result of filtering Fig. 4.38(a) in the frequency domain with the filter in (b). (d) Result of filtering the same image with the spatial filter in (a). The results are identical.

Filtro Passa-Baixas



a b c

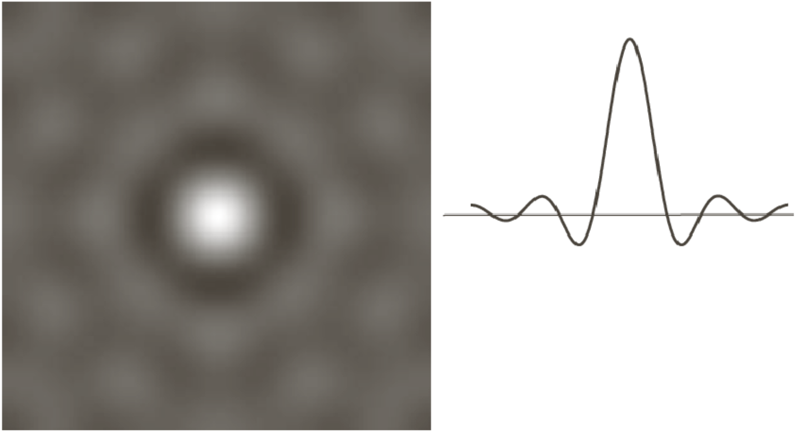
FIGURE 4.40 (a) Perspective plot of an ideal lowpass-filter transfer function. (b) Filter displayed as an im
(c) Filter radial cross section.



a b

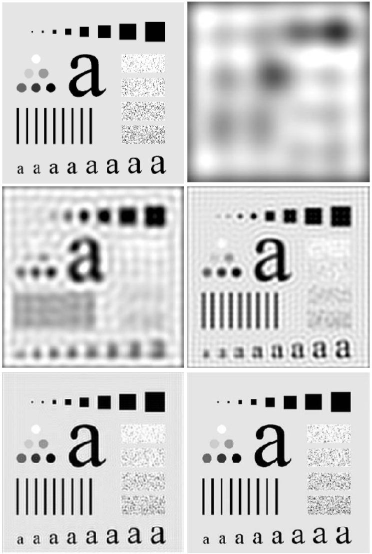
FIGURE 4.41 (a) Test pattern of size 688 × 688 pixels, and (b) its Fourier spectrum. The spectrum is double the image size due to padding but is shown in half size so that it fits in the page. The superimposed circles have radii equal to 10, 30, 60, 160, and 460 with respect to the full-size spectrum image. These radii enclose 87.0, 93.1, 95.7, 97.8, and 99.2% of the padded image power, respectively.

Filtro Ideal



a b

FIGURE 4.43 (a) Representation in the spatial domain of an ILPF of radius 5 and size 1000 × 1000. (b) Intensity profile of a horizontal line passing through the center of the image.



Filtro Ideal: Frequências de corte com valores de raio iguais a 10, 30, 60, 160 e 460. A potência removida por estes filtros é 13, 6.9, 4.3, 2.2, e 0.8% do total.

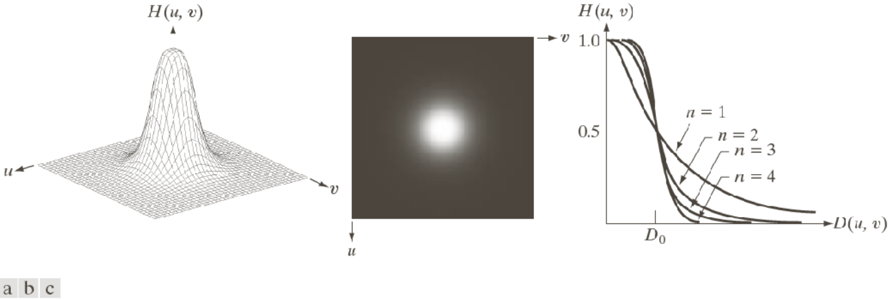
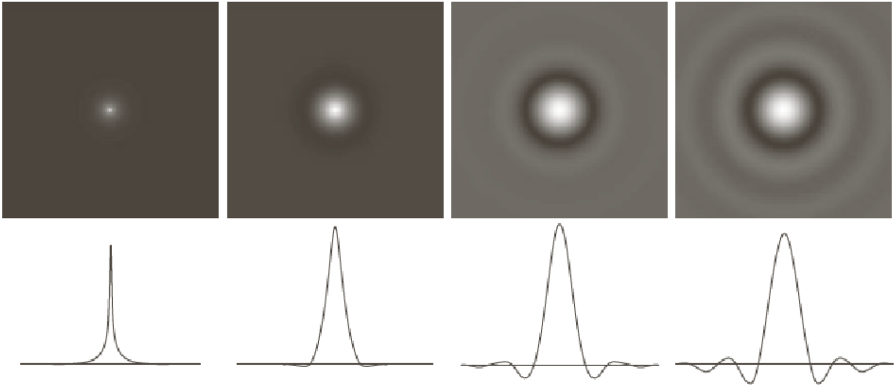


FIGURE 4.44 (a) Perspective plot of a Butterworth lowpass-filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.

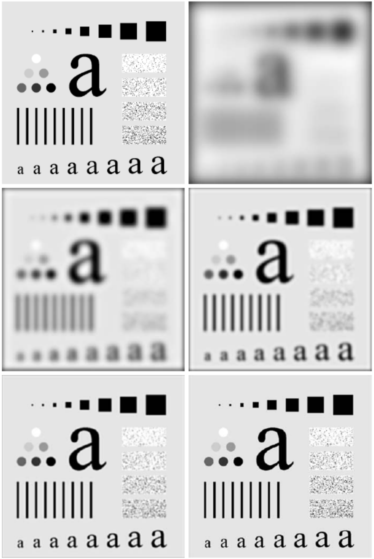
$$H(u,v) = \frac{1}{1 + [D(u,v)/D_o]^{2n}}$$

No tempo:



a b c d

FIGURE 4.46 (a)–(d) Spatial representation of BLPFs of order 1, 2, 5, and 20, and corresponding intensity profiles through the center of the filters (the size in all cases is 1000 $\times$ 1000 and the cutoff frequency is 5). Observe how ringing increases as a function of filter order.



Filtro Butterworth: Frequências de corte com valores de raio iguais a 10, 30, 60, 160 e 460. A potência removida por estes filtros é 13, 6.9, 4.3, 2.2, e 0.8% do total.

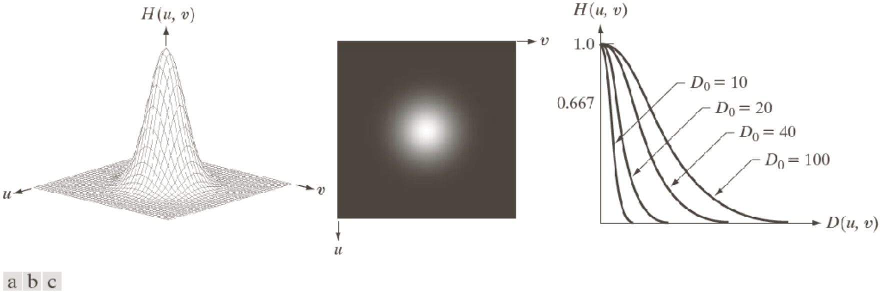
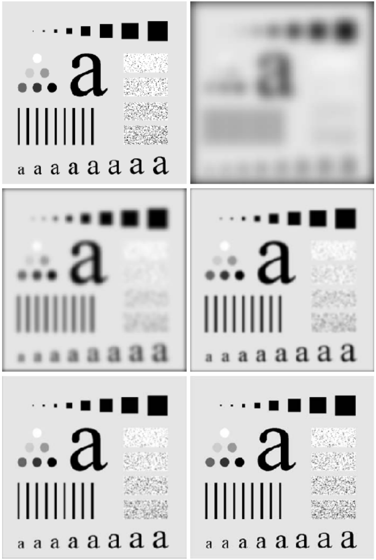


FIGURE 4.47 (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of D_0.

$$H(u,v) = e^{-D^2(u,v)/2D_o^2}$$

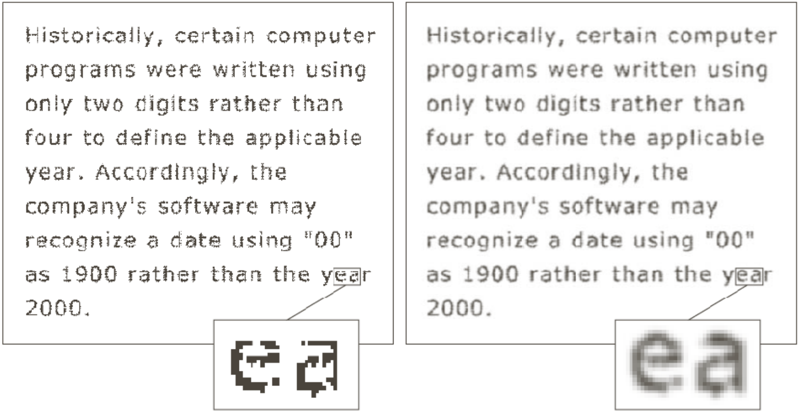


Filtro Gaussiano: Frequências de corte com valores de raio iguais a 10, 30, 60, 160 e 460. A potência removida por estes filtros é 13, 6.9, 4.3, 2.2, e 0.8% do total.

TABLE 4.4
Lowpass filters. D_0 is the cutoff frequency and n is the order of the Butterworth filter.

Ideal	Butterworth	Gaussian
$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \le D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases}$	$H(u,v) = \dfrac{1}{1 + [D(u,v)/D_0]^{2n}}$	$H(u,v) = e^{-D^2(u,v)/2D_0^2}$

Exemplos – Filtros



a b

FIGURE 4.49 (a) Sample text of low resolution (note broken characters in magnified view). (b) Result of filtering with a GLPF (broken character segments were joined).

Exemplos – Filtros

a b c

FIGURE 4.50 (a) Original image (784 × 732 pixels). (b) Result of filtering using a GLPF with $D_0 = 100$. (c) Result of filtering using a GLPF with $D_0 = 80$. Note the reduction in fine skin lines in the magnified sections in (b) and (c).

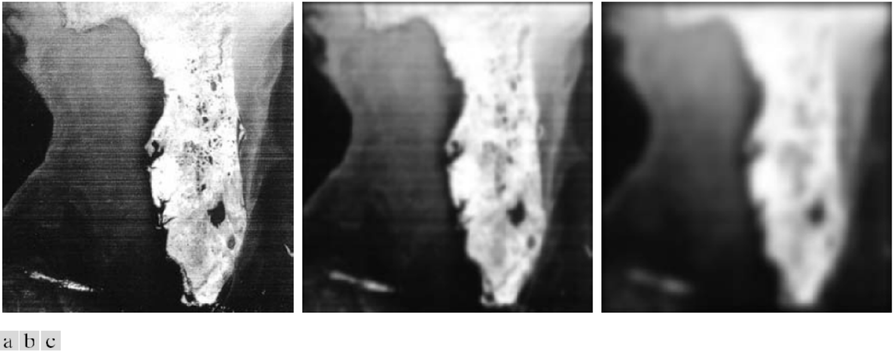


FIGURE 4.51 (a) Image showing prominent horizontal scan lines. (b) Result of filtering using a GLPF with $D_0 = 50$. (c) Result of using a GLPF with $D_0 = 20$. (Original image courtesy of NOAA.)

Filtros Passa-altas:

$$H_{HP}(u,v) = 1 - H_{LP}(u,v)$$

TABLE 4.5
Highpass filters. D_0 is the cutoff frequency and n is the order of the Butterworth filter.

Ideal	Butterworth	Gaussian
$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \leq D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases}$	$H(u,v) = \dfrac{1}{1 + [D_0/D(u,v)]^{2n}}$	$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$

Filtros Passa-Altas

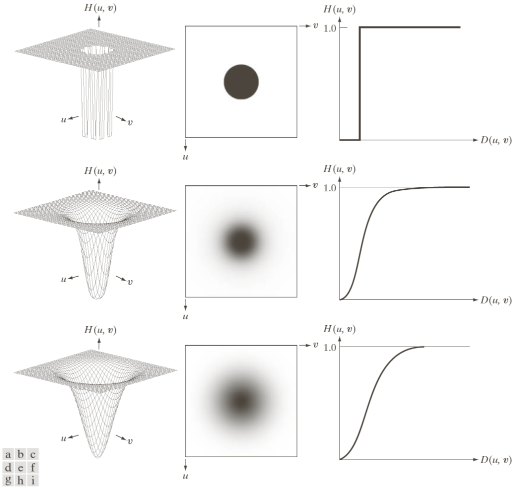


FIGURE 4.52 Top row: Perspective plot, image representation, and cross section of a typical ideal highpass filter. Middle and bottom rows: The same sequence for typical Butterworth and Gaussian highpass filters.

Ideal

Butterworth

Gaussiano

Representação Espacial

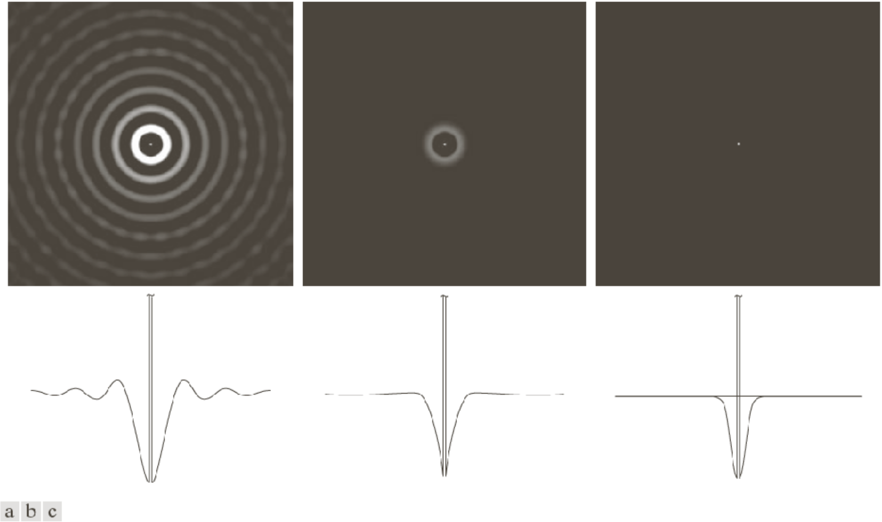


FIGURE 4.53 Spatial representation of typical (a) ideal, (b) Butterworth, and (c) Gaussian frequency domain highpass filters, and corresponding intensity profiles through their centers.

Passa-Altas Ideal

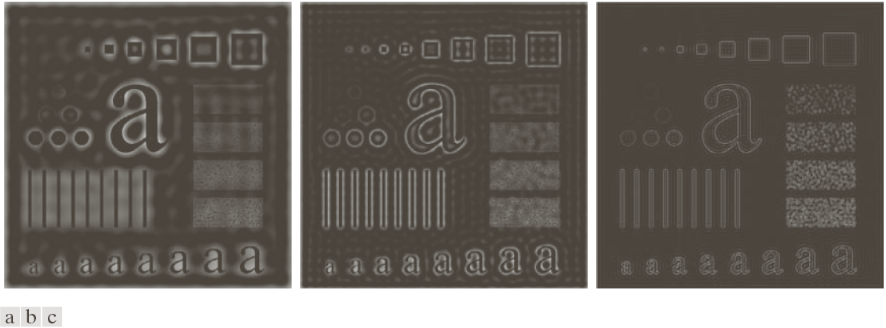


FIGURE 4.54 Results of highpass filtering the image in Fig. 4.41(a) using an IHPF with $D_0 = 30, 60$, and 160.

a b c

FIGURE 4.55 Results of highpass filtering the image in Fig. 4.41(a) using a BHPF of order 2 with $D_0 = 30, 60$, and 160, corresponding to the circles in Fig. 4.41(b). These results are much smoother than those obtained with an IHPF.

a b c

FIGURE 4.56 Results of highpass filtering the image in Fig. 4.41(a) using a GHPF with $D_0 = 30, 60$, and 160, corresponding to the circles in Fig. 4.41(b). Compare with Figs. 4.54 and 4.55.

Exemplo

a b c

FIGURE 4.57 (a) Thumb print. (b) Result of highpass filtering (a). (c) Result of thresholding (b). (Original image courtesy of the U.S. National Institute of Standards and Technology.)

Laplaciano

$$g(x,y) = f(x,y) - \nabla^2 f(x,y)$$
$$g(x,y) = F^{-1}\left\{\left[1 + (u - M/2)^2 (v - N/2)^2\right] F(u,v)\right\} f(x,y)$$

Exemplo

a b

FIGURE 4.58 (a) Original, blurry image. (b) Image enhanced using the Laplacian in the frequency domain. Compare with Fig. 3.38(e).

Lembrando

$$\boxed{g(x,y) = f(x,y) - f_{LP}(x,y)} \quad \boxed{f_{HB}(x,y) = Af(x,y) - f_{LP}(x,y)}$$

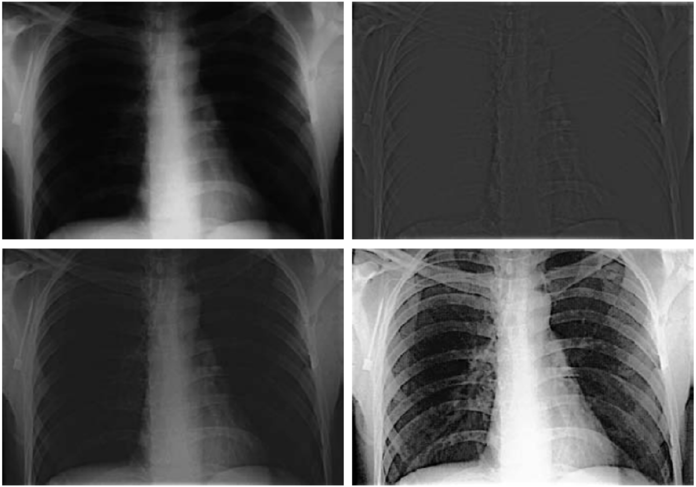
$$f_{HB}(x,y) = (A-1)f(x,y) + f(x,y) - f_{LP}(x,y)$$

e

$$f_{hb}(x,y) = (A-1)f(x,y) + f_{HP}(x,y)$$

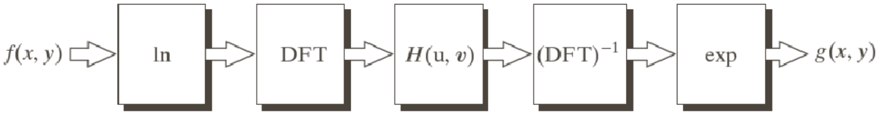
$$\boxed{H_{HP}(u,v) = 1 - H_{LP}(u,v)}$$

$$\boxed{H_{HB}(u,v) = (A-1) + H_{HP}(u,v)}$$



a	b
c	d

FIGURE 4.59 (a) A chest X-ray image. (b) Result of highpass filtering with a Gaussian filter. (c) Result of high-frequency-emphasis filtering using the same filter. (d) Result of performing histogram equalization on (c). (Original image courtesy of Dr. Thomas R.



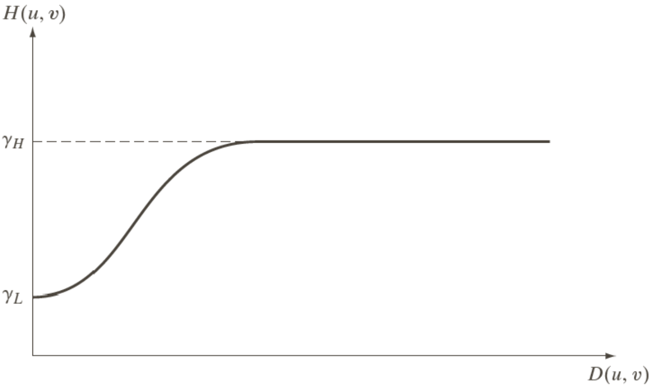


FIGURE 4.61 Radial cross section of a circularly symmetric homomorphic filter function. The vertical axis is at the center of the frequency rectangle and $D(u, v)$ is the distance from the center.

$$f(x,y) = i(x,y) \cdot r(x,y)$$

Mas,

$$\text{TF}\left\{f(x,y)\right\} \neq \text{TF}\left\{i(x,y)\right\} \cdot \text{TF}\left\{r(x,y)\right\}$$

$$f(x,y) = i(x,y) \cdot r(x,y)$$

Mas,

$$\text{TF}\{f(x,y)\} \neq \text{TF}\{i(x,y)\} \cdot \text{TF}\{r(x,y)\}$$

Considerando que

$$z(x,y) = \ln f(x,y) = \ln i(x,y) + \ln r(x,y),$$

temos que:

$$\text{TF}\{z(x,y)\} = \text{TF}\{\ln f(x,y)\} = \text{TF}\{\ln i(x,y)\} + \text{TF}\{\ln r(x,y)\}.$$

Logo

$$Z(u,v) = F_i(u,v) + F_r(u,v)$$

Filtragem Homomórfica

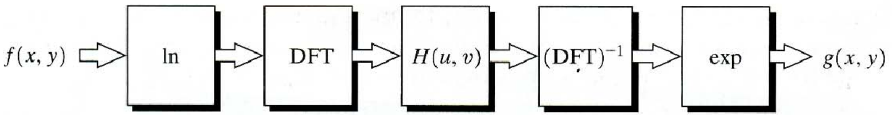
$$S(u,v) = H(u,v) \cdot Z(u,v)$$
$$= H(u,v)F_i(u,v) + H(u,v)F_r(u,v)$$

$$s(x,y) = \text{TF}^{-1}\{S(u,v)\}$$
$$= \text{TF}^{-1}\{H(u,v)F_i(u,v)\} + \text{TF}^{-1}\{H(u,v)F_r(u,v)\}$$

$$i'(x,y) = \text{TF}^{-1}\{H(u,v)F_i(u,v)\}$$
$$r'(x,y) = \text{TF}^{-1}\{H(u,v)F_r(u,v)\}$$

$$\boxed{s(x,y) = i'(x,y) + r'(x,y)}$$

$$
\begin{aligned}
g(x,y) &= e^{s(x,y)} \\
&= e^{i'(x,y)} \cdot e^{r'(x,y)} \\
&= i_0(x,y) \cdot r_0(x,y)
\end{aligned}
$$

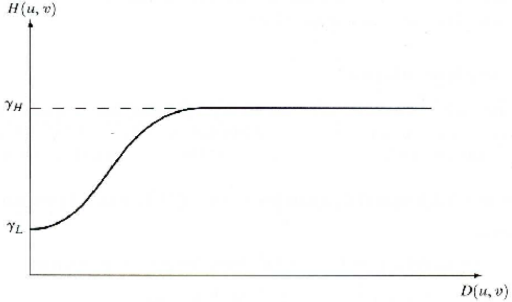


Filtragem Homomórfica

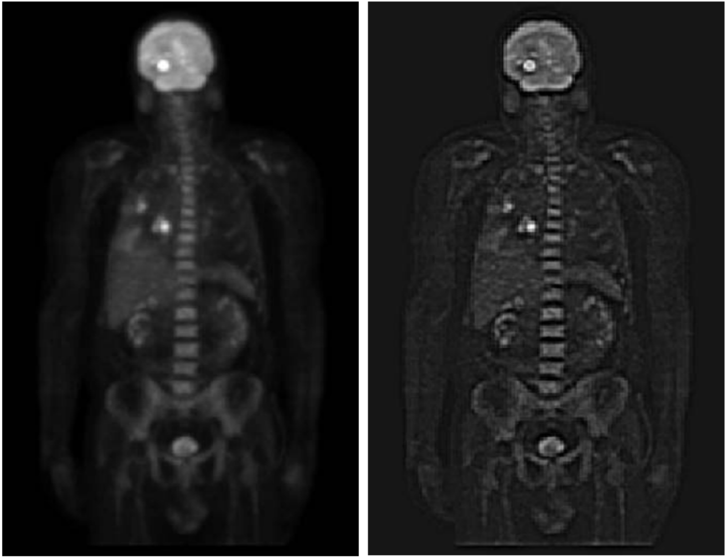
- Iluminação variações espaciais de baixa frequência
- Reflectância variações abruptas
- Filtro homomórfico trata diferentemente as baixas (i) e altas (r) frequências

$$H(u,v) = (\gamma_H - \gamma_L)\left[1 - e^{-c\left(D^2(u,v)/D_0^2\right)}\right] + \gamma_L$$

$\gamma_L < 1$ and $\gamma_H > 1$.



Exemplo: Filtragem Homomórfica



a b

FIGURE 4.62 (a) Full body PET scan. (b) Image enhanced using homomorphic filtering. (Original image courtesy of Dr. Michael E. Casey, CTI PET Systems.)

$$H_{BP}(u,v) = 1 - H_{BR}(u,v)$$

TABLE 4.6

Bandreject filters. W is the width of the band, D is the distance $D(u, v)$ from the center of the filter, D_0 is the cutoff frequency, and n is the order of the Butterworth filter. We show D instead of $D(u, v)$ to simplify the notation in the table.

Ideal	Butterworth	Gaussian
$H(u,v) = \begin{cases} 0 & \text{if } D_0 - \frac{W}{2} \leq D \leq D_0 + \frac{W}{2} \\ 1 & \text{otherwise} \end{cases}$	$H(u,v) = \dfrac{1}{1 + \left[\dfrac{DW}{D^2 - D_0^2}\right]^{2n}}$	$H(u,v) = 1 - e^{-\left[\frac{D^2 - D_0^2}{DW}\right]^2}$

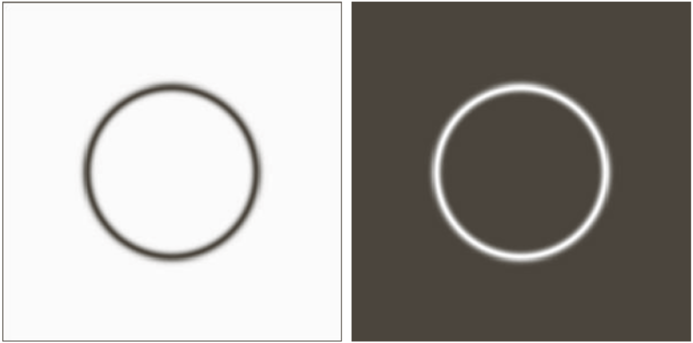


FIGURE 4.63
(a) Bandreject Gaussian filter. (b) Corresponding bandpass filter. The thin black border in (a) was added for clarity; it is not part of the data.

Filtros Notch

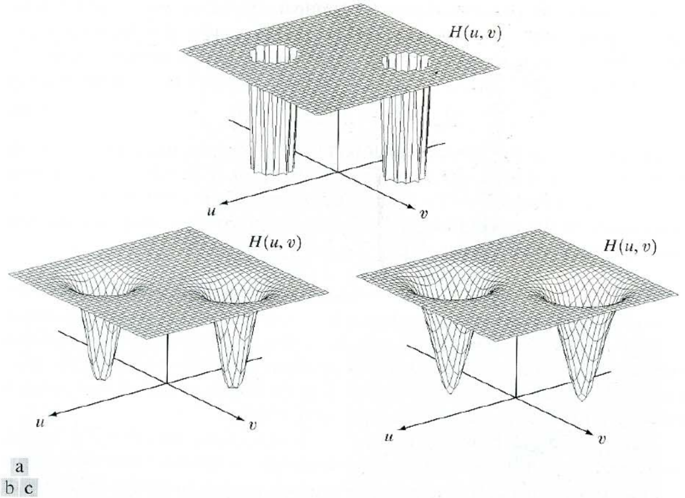


FIGURE 5.18 Perspective plots of (a) ideal, (b) Butterworth (of order 2), and (c) Gaussian notch (reject) filters.

Filtros Notch

- Ideal:

$$H(u,v) = \begin{cases} 0, & \text{se } D_1(u,v) \leq D_0 \text{ ou } D_2(u,v) \leq D_0 \\ 1, & \text{caso contrário} \end{cases}$$

$$D_1(u,v) = \left[(u - M/2 - u_0)^2 + (v - N/2 - v_0)^2\right]^{1/2}$$

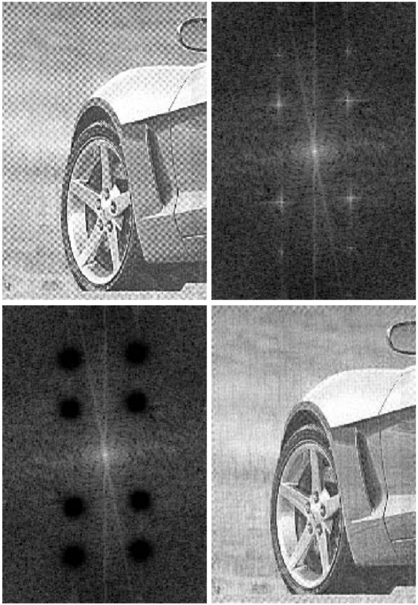
$$D_2(u,v) = \left[(u - M/2 + u_0)^2 + (v - N/2 + v_0)^2\right]^{1/2}$$

- Butterworth:

$$H(u,v) = \frac{1}{1 + \left[\frac{D_0^2}{D_1(u,v)D_2(u,v)}\right]^n}$$

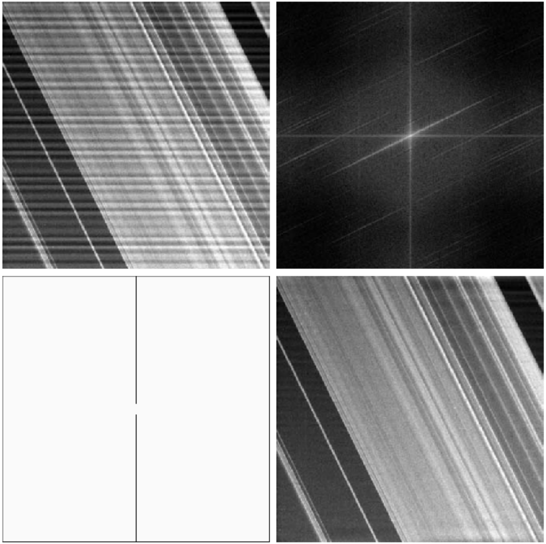
- Gaussiano:

$$H(u,v) = 1 - e^{-\frac{1}{2}\left[\frac{D_1(u,v)D_2(u,v)}{D_0^2}\right]}$$



a b
c d

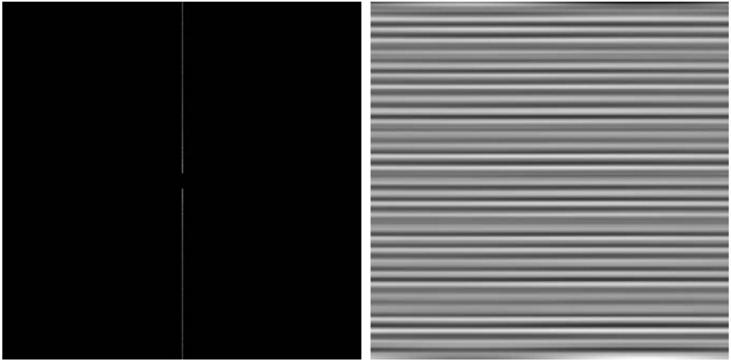
FIGURE 4.64 (a) Sampled newspaper image showing a moiré pattern. (b) Spectrum. (c) Butterworth notch reject filter multiplied by the Fourier transform. (d) Filtered image.



a b
c d

FIGURE 4.65 (a) 674 × 674 image of the Saturn rings showing nearly periodic interference. (b) Spectrum: The bursts of energy in the vertical axis near the origin correspond to the interference pattern. (c) A vertical notch reject filter. (d) Result of filtering. The thin black border in (c) was added for clarity; it is not part of the data. (Original image courtesy of Dr. Robert A. West, NASA/JPL.)

Filtros Notch



a b

FIGURE 4.66 (a) Result (spectrum) of applying a notch pass filter to the DFT of Fig. 4.65(a). (b) Spatial pattern obtained by computing the IDFT of (a).