

Processamento de Imagens

Introdução

Mylène Christine Queiroz de Farias

Departamento de Engenharia Elétrica

Universidade de Brasília (UnB)

Brasília, DF 70910-900

`mylene@unb.br`

22 de Março de 2016

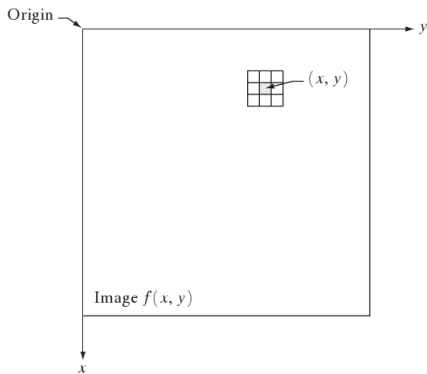
Aula 03: Capítulo 3 – Transformações Espaciais



$$g(x, y) = T[f(x, y)]$$

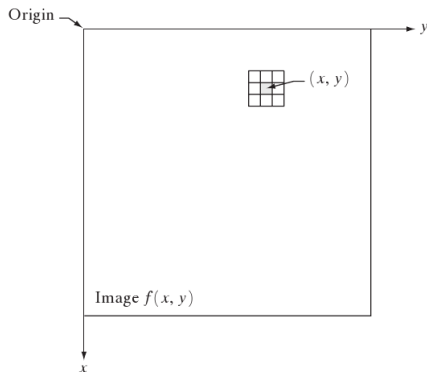
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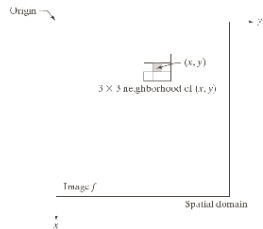
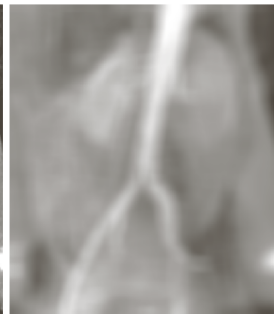
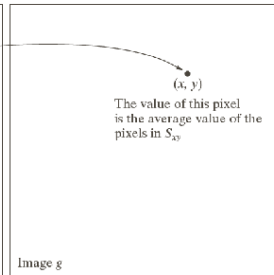
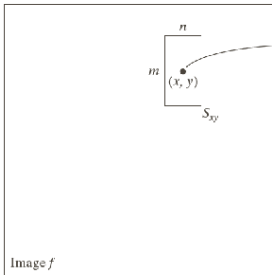
$$H(a \cdot f + b \cdot g) = a \cdot H(f) + b \cdot H(g)$$



$$g(x, y) = T[f(x, y)]$$

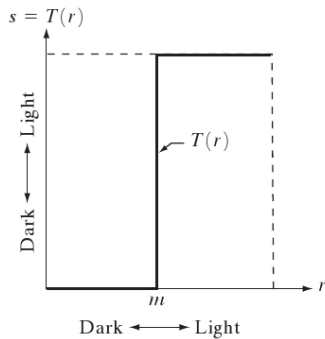
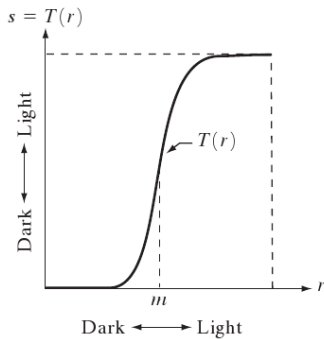
- Global: Toda a imagem
- Vizinhança: regiões (quadrados, círculos, etc.)
- Pixel-a-pixel: 1×1



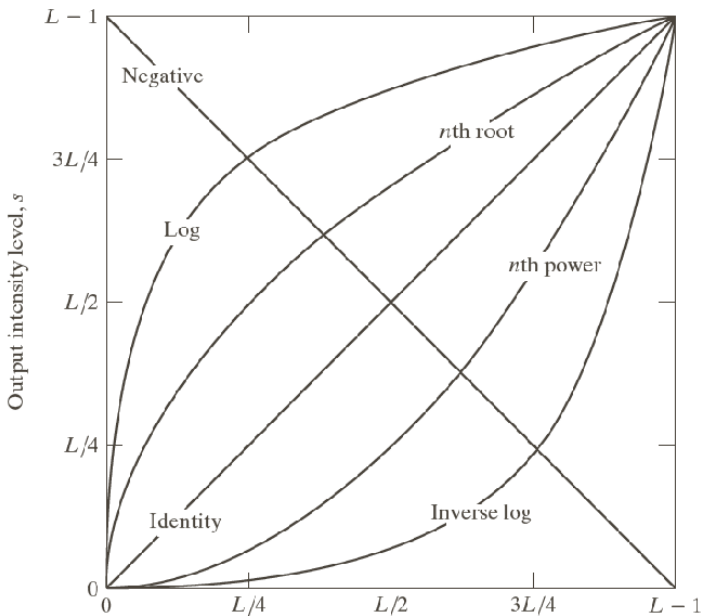


Transformação

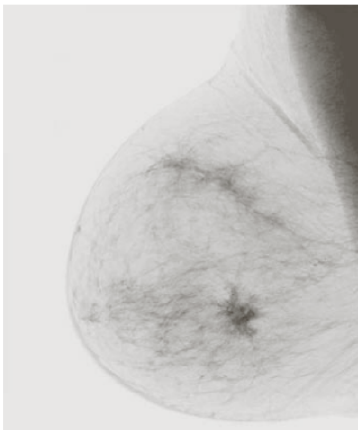
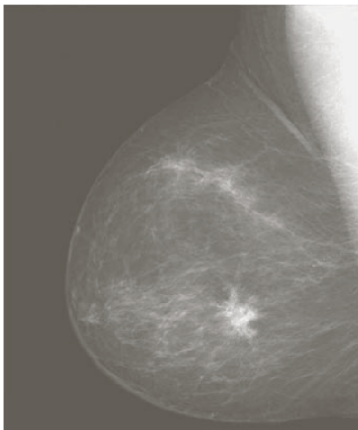
$$s = T(r)$$



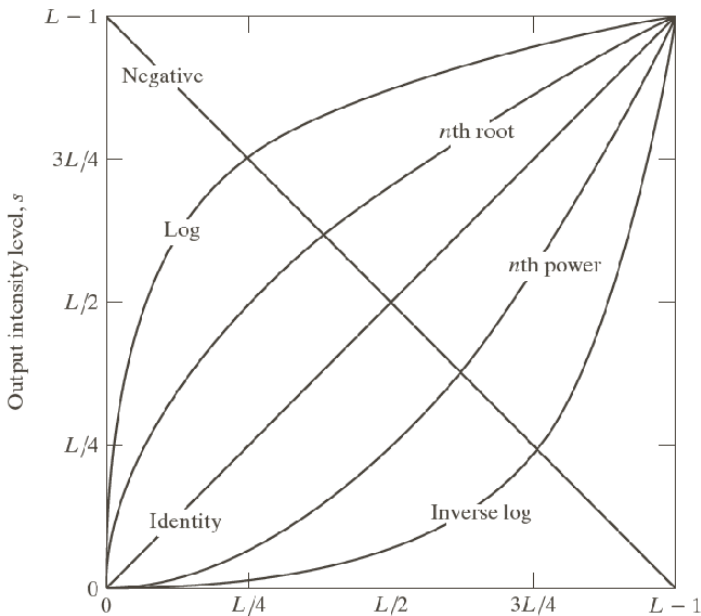
Transformação



$$s = L - 1 - r$$



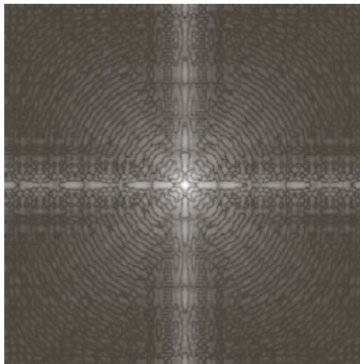
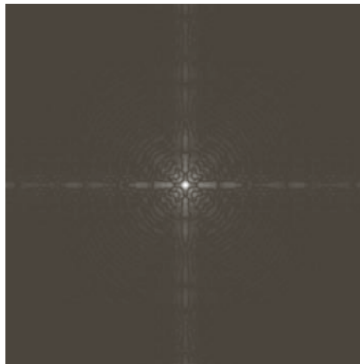
Transformação



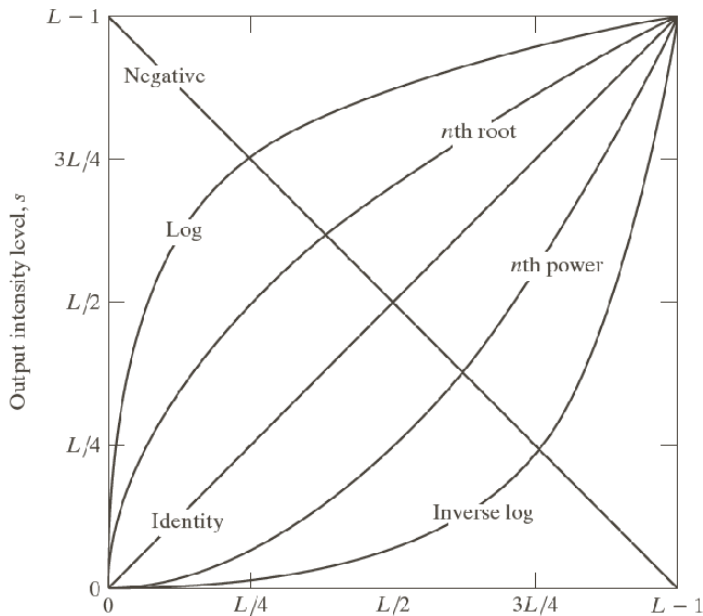
$$s = c \log(1 + r)$$

a b

FIGURE 3.5
(a) Fourier spectrum.
(b) Result of applying the log transformation in Eq. (3.2-2) with $c = 1$.



Transformação



$$s = cr^\gamma$$

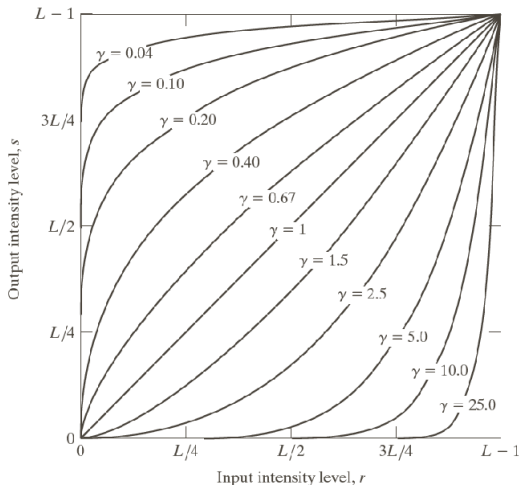
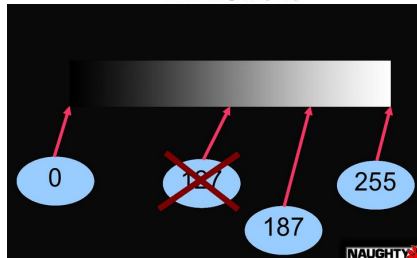
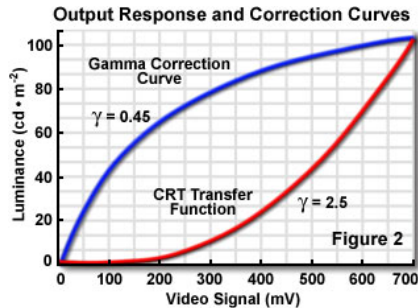
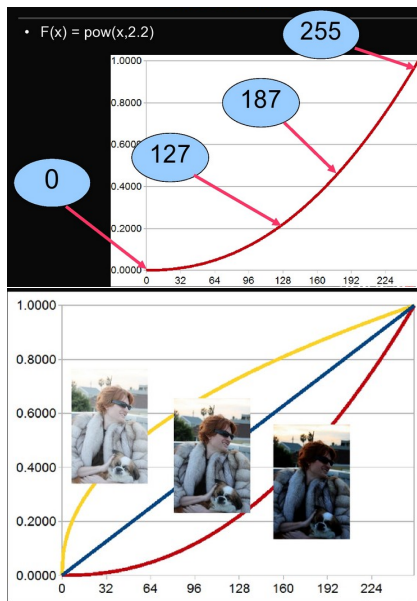


FIGURE 3.6 Plots of the equation $s = cr^\gamma$ for various values of γ ($c = 1$ in all cases). All curves were scaled to fit in the range shown.

Função Gamma



Função Gamma



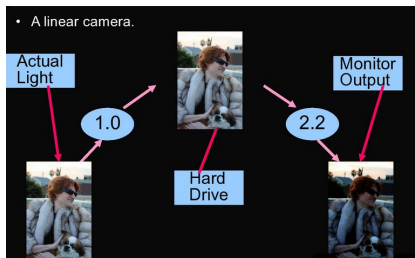
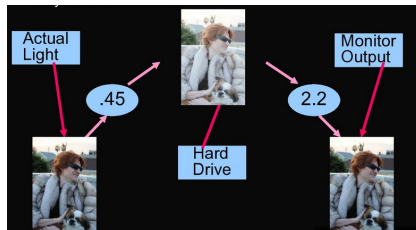


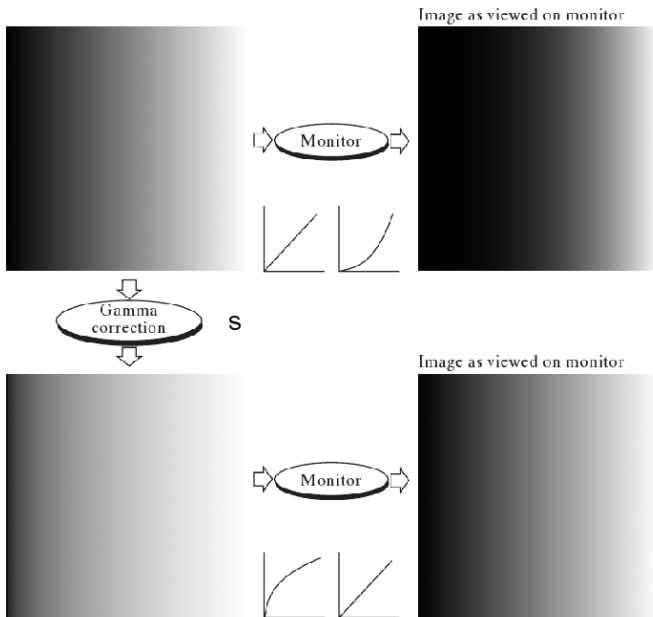
Imagem corrigida antes da visualização

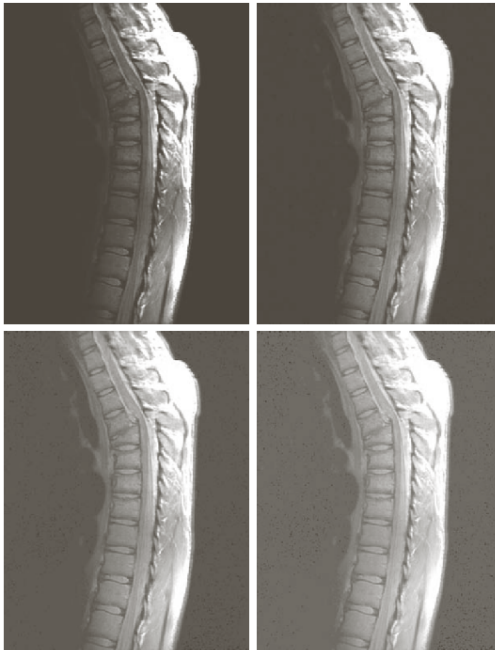


a	b
c	d

FIGURE 3.7

(a) Linear-wedge gray-scale image.
 (b) Response of monitor to linear wedge.
 (c) Gamma-corrected wedge.
 (d) Output of monitor.





a b
c d

FIGURE 3.8

(a) Magnetic resonance image (MRI) of a fractured human spine.

(b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and

$\gamma = 0.6, 0.4,$ and 0.3 , respectively.

(Original image courtesy of Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)

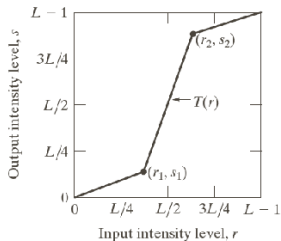


a	b
c	d

FIGURE 3.9

(a) Aerial image.
 (b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 3.0, 4.0$, and 5.0 , respectively. (Original image for this example courtesy of NASA.)

Função Lineares por partes



a b
c d

FIGURE 3.10

Contrast stretching.

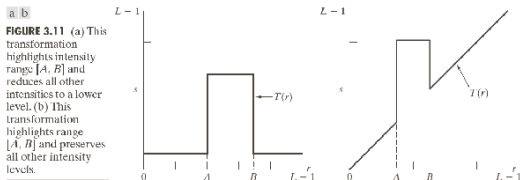
(a) Form of transformation function. (b) A low-contrast image.

(c) Result of contrast stretching.

(d) Result of thresholding.

(Original image courtesy of Dr.

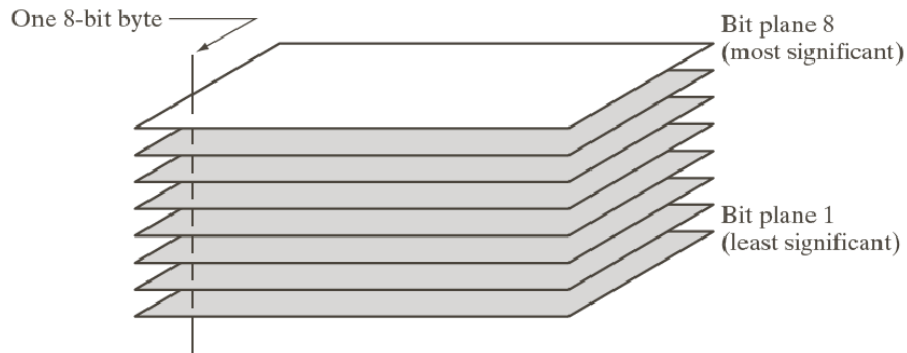
Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)



a b c

FIGURE 3.12 (a) Aortic angiogram. (b) Result of using a slicing transformation of the type illustrated in Fig. 3.11(a), with the range of intensities of interest selected in the upper end of the gray scale. (c) Result of using the transformation in Fig. 3.11(b), with the selected area set to black, so that grays in the area of the blood vessels and kidneys were preserved. (Original image courtesy of Dr. Thomas R. Gest, University of Michigan Medical School.)

Fatiamento por Planos de Bits



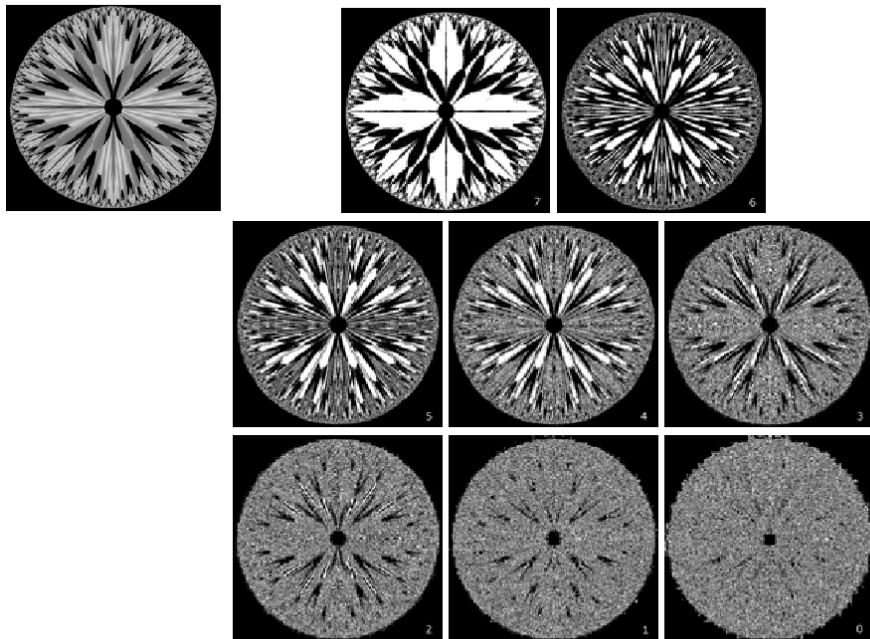


FIGURE 3.14 The eight bit planes of the image in Fig. 3.13. The number at the bottom, right of each image identifies the bit plane.



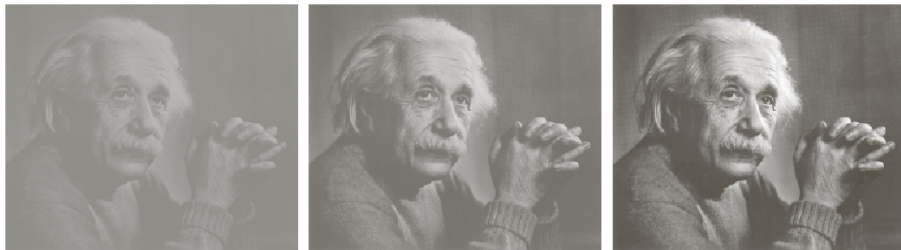
a	b	c
d	e	f
g	h	i

FIGURE 3.14 (a) An 8-bit gray-scale image of size 500×1192 pixels. (b) through (i) Bit planes 1 through 8, with bit plane 1 corresponding to the least significant bit. Each bit plane is a binary image.



a b c

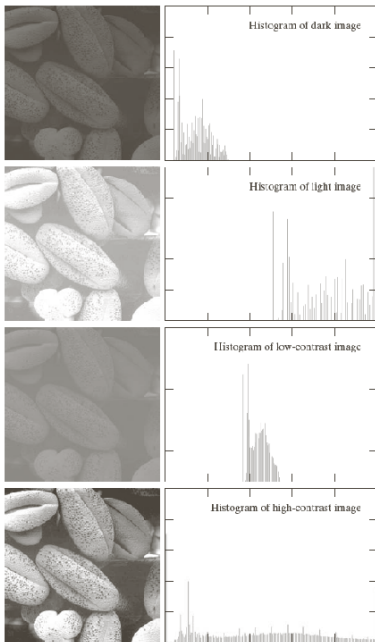
FIGURE 3.15 Images reconstructed using (a) bit planes 8 and 7; (b) bit planes 8, 7, and 6; and (c) bit planes 8, 7, 6, and 5. Compare (c) with Fig. 3.14(a).



a b c

FIGURE 2.41

Images exhibiting
(a) low contrast,
(b) medium
contrast, and
(c) high contrast.



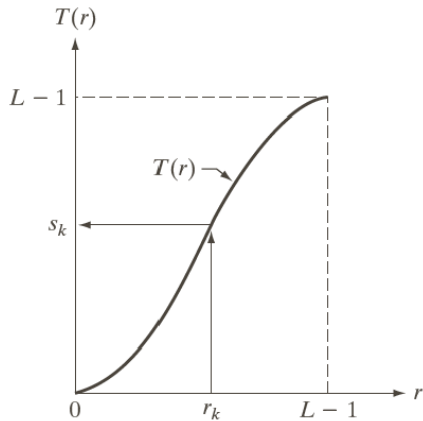
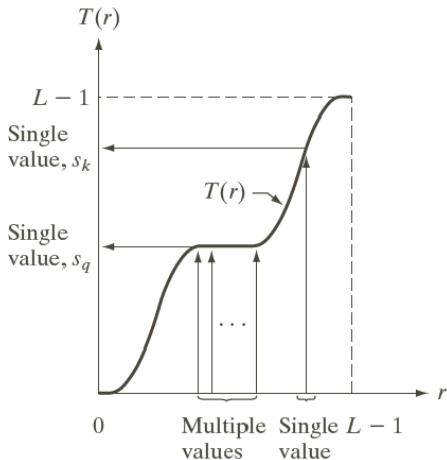
- Função discreta

$$h(r_k) = n_k$$

- r_k – nível
- n_k – número de pixels com nível r_k
- Normalizado
 - total de pixels
 - probabilidade
 - soma = 1

- A função de transformação de histograma permite uma distribuição de pixels seja transformada em outra mais “interessante”
- Utiliza-se uma função $T(r)$ que deve obedecer as seguintes condições:
 - 1 Ser monotonicamente crescente no intervalo de $0 \leq r \leq (L - 1)$
 - 2 $0 \leq T(r) \leq L - 1$ para $0 \leq r \leq (L - 1)$

Transformação de Histogramas



Transformação de Histogramas

- p_r e p_s são funções densidade de probabilidade(PDF);
- $T(r)$ é a função transformação de uma variável (r) em outra variável (s);

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$$p_s(s) = p_r(r) \left| \frac{dr}{ds} \right|$$

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$$p_s(s) = p_r(r) \left| \frac{dr}{ds} \right|$$

- Exemplo de $T(r)$:

$$s = T(r) = (L - 1) \int_0^r p_r(w) dw$$

Considerando:

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Transformação de Histogramas

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$$\frac{ds}{dr} = (L - 1) \frac{d}{dr} \int_0^r p_r(w) dw = (L - 1) p_r(r)$$

Transformação de Histogramas

Considerando:

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$$\frac{ds}{dr} = \frac{dT(r)}{dr}$$

$$\frac{ds}{dr} = (L - 1) \frac{d}{dr} \int_0^r p_r(w) dw = (L - 1) p_r(r)$$

$$p_s(s) = p_r(r) \left| \frac{dr}{ds} \right| = p_r(r) \left| \frac{1}{p_r(r)(L - 1)} \right| = \frac{1}{L - 1}$$

Distribuição Uniforme

Transformação de Histogramas

$$s = T(r) = (L - 1) \int_0^r p_r(w) dw$$

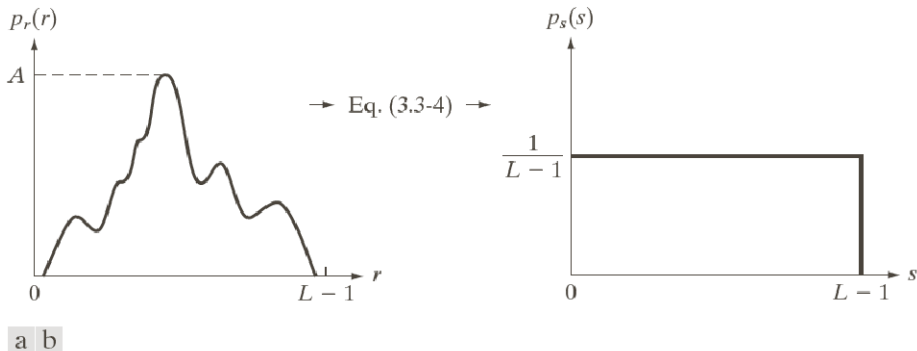


FIGURE 3.18 (a) An arbitrary PDF. (b) Result of applying the transformation in Eq. (3.3-4) to all intensity levels, r . The resulting intensities, s , have a uniform PDF, independently of the form of the PDF of the r 's.

- Caso Contínuo

$$s = T(r) = (L - 1) \int_0^r p_r(w) dw$$

Transformação de Histogramas

- Caso Contínuo

$$s = T(r) = (L - 1) \int_0^r p_r(w) dw$$

- Caso Discreto

Para $k = 0, 1, \dots, L - 1$.

$$\begin{aligned} s_k = T(r_k) &= (L - 1) \sum_{j=0}^k p_r(r_j) s_k \\ &= (L - 1) \sum_{j=0}^k \frac{n_j}{M \cdot N} \\ &= \frac{(L - 1)}{M \cdot N} \sum_{j=0}^k n_j \end{aligned}$$

Transformação de Histogramas

Exemplo: Imagem de 3 bits ($L = 8$) de 64×64 ($M \cdot N = 4096$)

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

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$$s_0 = T(r_0) = 7 \sum_{j=0}^0 p_r(r_j) = 7 \cdot 0,19 = 1,33 \longrightarrow 1$$

$$s_1 = T(r_1) = 7 \sum_{j=0}^1 p_r(r_j) = 7 \cdot (0,19 + 0,25) = 3,08 \longrightarrow 3$$

$$s_2 = 4,55 \longrightarrow 5 \quad s_3 = 5,67 \longrightarrow 6$$

$$s_4 = 6,23 \longrightarrow 6 \quad s_5 = 6,65 \longrightarrow 7$$

$$s_6 = 6,86 \longrightarrow 7 \quad s_7 = 7,00 \longrightarrow 7$$

Transformação de Histogramas

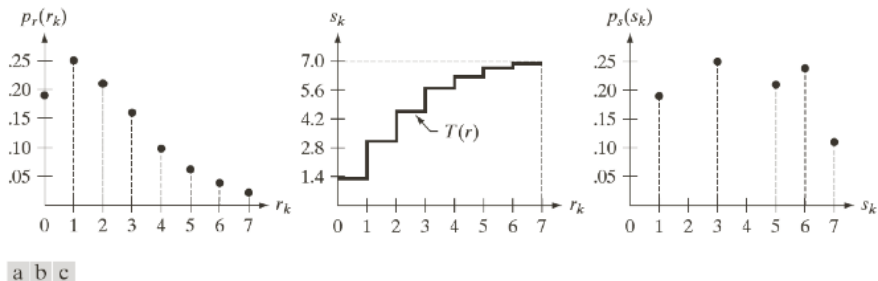


FIGURE 3.19 Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

Transformação de Histogramas

$$s_k = T(r_k)$$
$$= \frac{(L-1)}{M \cdot N} \sum_{j=0}^k n_j$$

r_k	n_k	$p_r(r_k) = n_k/MN$
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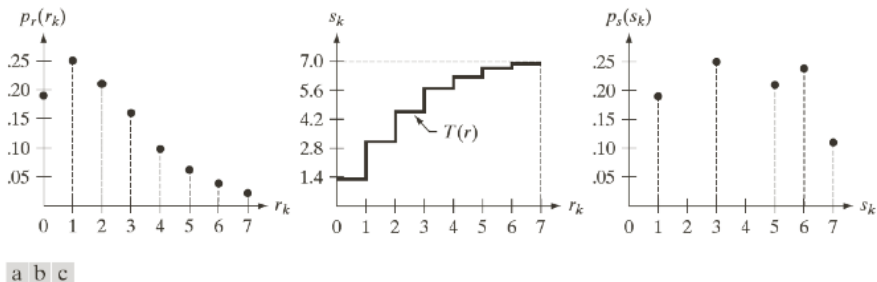
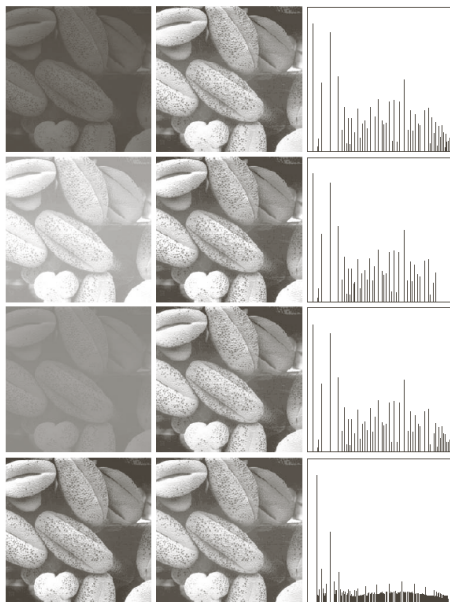
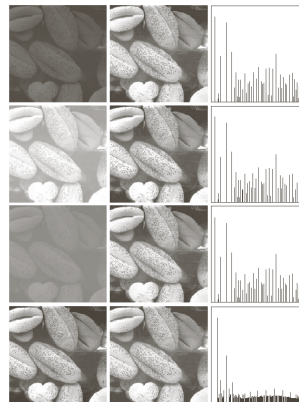
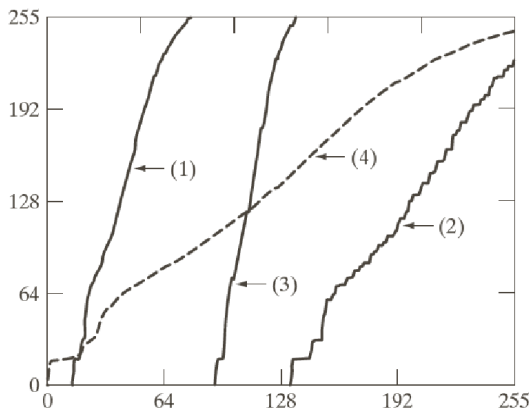


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Pergunta:

A equalização de histograma dá sempre bons resultados?

Pergunta:

A equalização de histograma dá sempre bons resultados? NÃO.

- Uma imagem tem um histograma específico
- Seja $p_r(r)$ e $p_z(z)$ as funções de densidades de probabilidade contínuas das variáveis r e z .
- $p_z(z)$ é a função densidade de probabilidade *especificada*
- Seja s uma variável aleatória

$$s = T(r) = (L - 1) \int_0^r p_r(w) dw$$

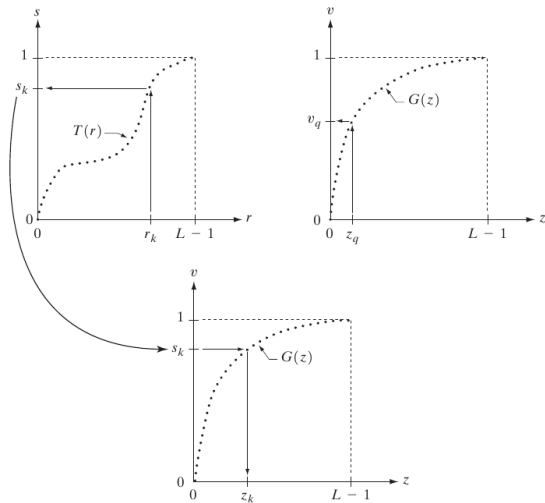
- Obtemos uma função G :

$$s = T(r) = (L - 1) \int_0^r p_r(w) dw$$

$$G(z) = (L - 1) \int_0^z p_z(t) dt = s$$

$$z = G^{-1}(s) = G^{-1}[T(r)]$$

Especificação de Histogramas



- 1 Obter $p_r(r)$ da imagem de entrada e calcular os valores de s :

$$s = T(r) = (L - 1) \int_0^r p_r(w) dw$$

- 2 Usar a PDF especificada para obter a função $G(z)$:

$$G(z) = (L - 1) \int_0^z p_z(t) dt = s$$

- 3 Mapear de s para z

$$z = G^{-1}(s)$$

- **Exemplo:** Assumindo valores de intensidade contínua:

$$p_r(r) = \begin{cases} \frac{2r}{(L-1)^2}, & \text{para } 0 \leq r \leq L-1 \\ 0, & \text{caso contrário} \end{cases} \quad (1)$$

- Encontre a transformação que produzirá:

$$p_z(z) = \begin{cases} \frac{3z^2}{(L-1)^3}, & \text{para } 0 \leq z \leq L-1 \\ 0, & \text{caso contrário} \end{cases} \quad (2)$$

Passos:

- Encontre a transformação da equalização:

$$\begin{aligned}s &= T(r) = (L - 1) \int_0^r p_r(w) dw \\ &= (L - 1) \int_0^r \frac{2w}{(L - 1)^2} dw = \frac{r^2}{L - 1}\end{aligned}$$

Passos:

- Encontre a transformação da equalização:

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- A transformação para o histograma:

$$\begin{aligned}G(z) &= (L - 1) \int_0^z p_z(t) dt \\ &= (L - 1) \int_0^z \frac{3t^2}{(L - 1)^3} dt = \frac{z^3}{(L - 1)^2}\end{aligned}$$

- A função transformação:

$$\begin{aligned} z &= [(L-1)^2 s]^{1/3} = \left[(L-1)^2 \frac{r^2}{L-1} \right]^{1/3} \\ &= [(L-1)r^2]^{1/3} \end{aligned}$$

- Logo, para obter a imagem transformada, aplica-se a função acima na variável de entrada r .

Caso discreto:

- Obter $p_r(r_j)$ da imagem de entrada e depois os valores de s_k , arredondando os valores para os inteiros no intervalo $[0, L - 1]$:

$$s_k = T(r_k) = \frac{(L - 1)}{M \cdot N} \sum_{j=0}^k n_j$$

- Usar a PDF especificada para obter a função $G(z_q)$, arredondando os valores para os inteiros no intervalo $[0, L - 1]$:

$$G(z_k) = (L - 1) \sum_{i=0}^q p_z(z_i) = s_k$$

- Mapear os valores de s_k para z_q :

$$z_q = G^{-1}(s_k)$$

- Na prática, não se calcula a inversa...
- Calcula-se todos os possíveis valores de G para q

$$G(z_k) = (L - 1) \sum_{i=0}^q p_z(z_i) = s_k$$

- Estes valores são escalonados e arredondados aos inteiros mais próximos no intervalo $[0, L - 1]$ e armazenados (tabela)
- Para um dado valor de s_k , encontramos o valor mais próximo nesta tabela.

Caso discreto:

- 1 Calcule o histograma $p_r(r)$ da imagem de entrada e use-o para calcular a transformação de equalização:

$$s_k = T(r_k) = \frac{(L-1)}{M \cdot N} \sum_{j=0}^k n_j$$

Arredonde os resultados, s_k para inteiros no intervalos $[0, L-1$

- 2 Calcule todos os valores da transformação G para $q = 0, 1, \dots, L-1$

$$G(z_k) = (L-1) \sum_{i=0}^q p_z(z_i) = s_k$$

Arredonde os resultados de G para inteiros no intervalos $[0, L-1$ e armazene estes resultados em uma tabela.

Caso discreto:

- 1 Para cada valor de s_k , $k = 0, 1, \dots, L - 1$, utilize os valores de G (passo 2) para encontrar o valor correspondente de z_q de forma que $G(z_q)$ esteja o mais próximo de s_k . Armazene estes valores de mapeamento de s para z .
- 2 Construa a imagem com histograma especificado realizando primeiramente a equalização e depois o mapeamento de s_k em z_q .

Exemplo:

- Imagem de 3 bits ($L = 8$), 64×64 ($M \cdot N = 4096$)

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

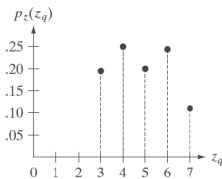
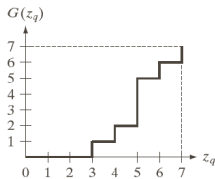
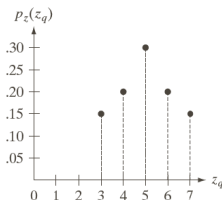
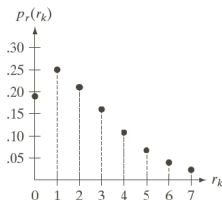
z_q	Specified $p_z(z_q)$	Actual $p_z(z_k)$
$z_0 = 0$	0.00	0.00
$z_1 = 1$	0.00	0.00
$z_2 = 2$	0.00	0.00
$z_3 = 3$	0.15	0.19
$z_4 = 4$	0.20	0.25
$z_5 = 5$	0.30	0.21
$z_6 = 6$	0.20	0.24
$z_7 = 7$	0.15	0.11

Especificação de Histogramas

Exemplo:

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

z_q	Specified $p_z(z_q)$	Actual $p_z(z_k)$
$z_0 = 0$	0.00	0.00
$z_1 = 1$	0.00	0.00
$z_2 = 2$	0.00	0.00
$z_3 = 3$	0.15	0.19
$z_4 = 4$	0.20	0.25
$z_5 = 5$	0.30	0.21
$z_6 = 6$	0.20	0.24
$z_7 = 7$	0.15	0.11



Exemplo:

- Obter as amostras equalizadas:

$$s_0 = 1,33 \longrightarrow 1 \quad s_1 = 3,08 \longrightarrow 3$$

$$s_2 = 4,55 \longrightarrow 5 \quad s_3 = 5,67 \longrightarrow 6$$

$$s_4 = 6,23 \longrightarrow 6 \quad s_5 = 6,65 \longrightarrow 7$$

$$s_6 = 6,86 \longrightarrow 7 \quad s_7 = 7,00 \longrightarrow 7$$

- Calcular o valor das transformações:

$$G(z_0) = 7 \sum_{j=0}^0 p_z(z_j) = 0,00 \longrightarrow 0$$

$$G(z_1) = 0,00 \quad G(z_2) = 0,00$$

$$G(z_3) = 1,05 \quad G(z_4) = 2,45$$

$$G(z_5) = 4,55 \quad G(z_6) = 5,95$$

$$G(z_7) = 7,00$$

Exemplo:

- Obter as amostras equalizadas:

$$s_0 = 1,33 \longrightarrow 1 \quad s_1 = 3,08 \longrightarrow 3$$

$$s_2 = 4,55 \longrightarrow 5 \quad s_3 = 5,67 \longrightarrow 6$$

$$s_4 = 6,23 \longrightarrow 6 \quad s_5 = 6,65 \longrightarrow 7$$

$$s_6 = 6,86 \longrightarrow 7 \quad s_7 = 7,00 \longrightarrow 7$$

- Calcular o valor das transformações e **aproximar para os inteiros mais próximos**:

$$G(z_0) = 7 \sum_{j=0}^0 p_z(z_j) = 0,00 \longrightarrow 0$$

$$G(z_1) = 0,00 \longrightarrow 0 \quad G(z_2) = 0,00 \longrightarrow 0$$

$$G(z_3) = 1,05 \longrightarrow 1 \quad G(z_4) = 2,45 \longrightarrow 2$$

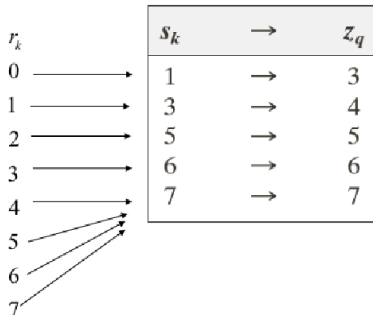
$$G(z_5) = 4,55 \longrightarrow 5 \quad G(z_6) = 5,95 \longrightarrow 6$$

$$G(z_7) = 7,00 \longrightarrow 7$$

Especificação de Histogramas

$$s_0 = 1, s_1 = 3, s_2 = 5, s_3 = 6, s_4 = 7, \\ s_5 = 7, s_6 = 7, s_7 = 7.$$

z_q	$G(z_q)$
$z_0 = 0$	0
$z_1 = 1$	0
$z_2 = 2$	0
$z_3 = 3$	1
$z_4 = 4$	2
$z_5 = 5$	5
$z_6 = 6$	6
$z_7 = 7$	7



Encontrar o menor valor de z_q de forma que $G(z_q)$ é mais próximo a s_k .

$$r_k \rightarrow z_q$$

$$0 \rightarrow 3$$

$$1 \rightarrow 4$$

$$2 \rightarrow 5$$

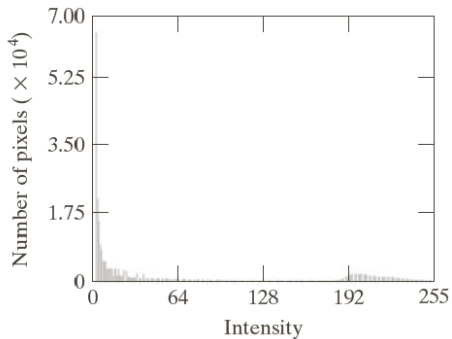
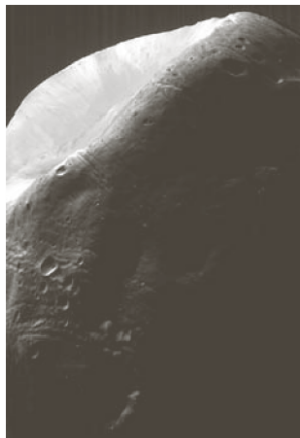
$$3 \rightarrow 6$$

$$4 \rightarrow 7$$

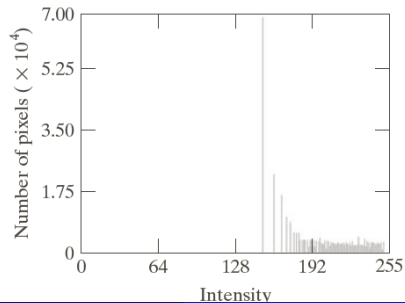
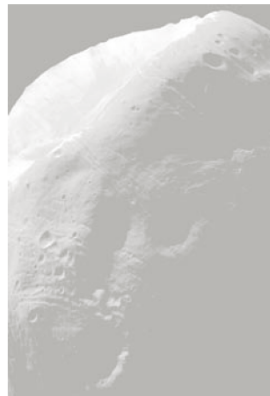
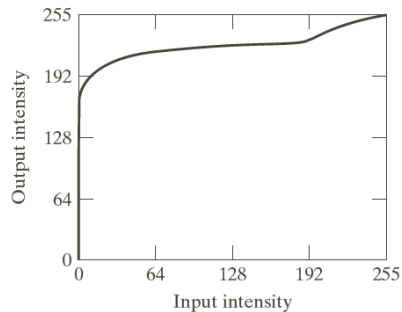
$$5 \rightarrow 7$$

$$6 \rightarrow 7$$

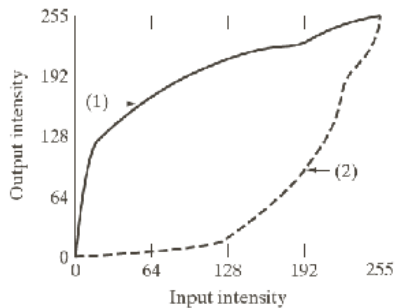
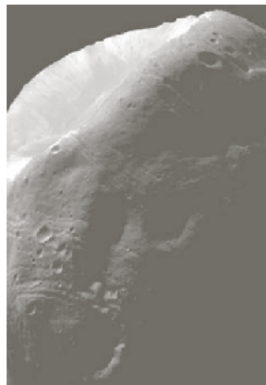
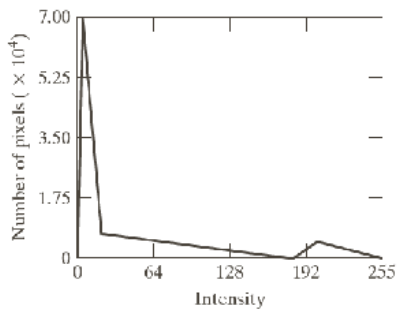
$$7 \rightarrow 7$$



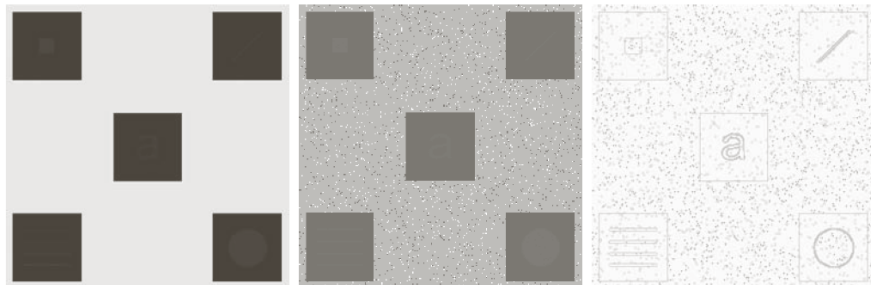
Equalizado



Especificado



- Definir uma vizinhança e mover o seu centro pixel a pixel
- Em cada posição, o histograma dos pontos é calculado. Tanto uma equalização de histograma ou uma especificação de histograma pode ser realizada
- Mapear a intensidade do pixel centralizado na vizinhança
- Mover-se para a próxima posição e repetir o procedimento.



(a) (b) (c)

FIGURE 3.26 (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization applied to (a), using a neighborhood of size 3×3 .

- Média da Intensidade

$$m = \sum_{i=0}^{L-1} r_i p(r_i) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y)$$

- Variância

$$\sigma^2 = u_2(r) = \sum_{i=0}^{L-1} (r_i - m)^2 p(r_i) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [f(x, y) - m]^2$$

- n-ésimo momento

$$u_n(r) = \sum_{i=0}^{L-1} (r_i - m)^n p(r_i)$$

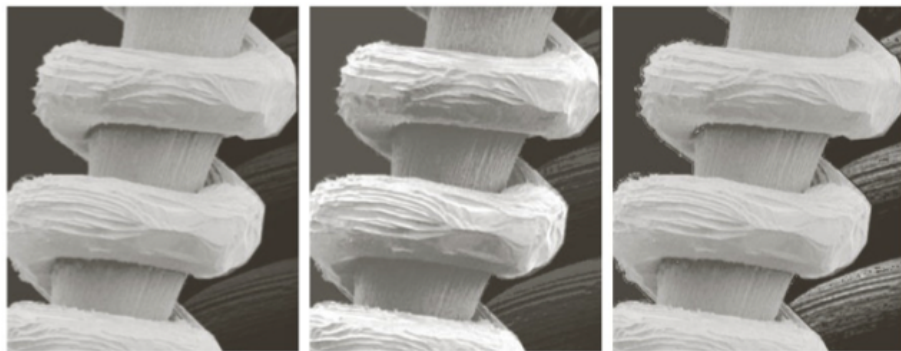
- Médias locais da Intensidade

$$m_{S_{xy}} = \sum_{i=0}^{L-1} r_i p_{S_{xy}}(r_i)$$

- Variância Local

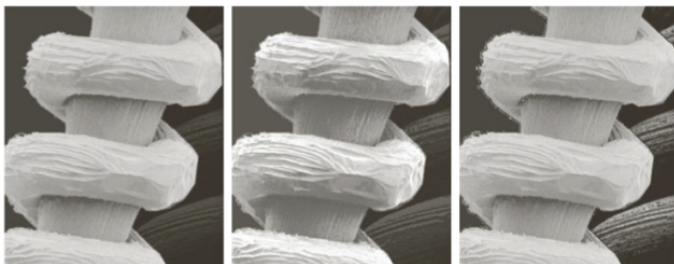
$$\sigma_{S_{xy}}^2 = \sum_{i=0}^{L-1} (r_i - m_{S_{xy}})^2 p_{S_{xy}}(r_i)$$

S_{xy} – vizinhança.



a b c

FIGURE 3.27 (a) SEM image of a tungsten filament magnified approximately 130 $\times$. (b) Result of global histogram equalization. (c) Image enhanced using local histogram statistics. (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)



a b c

FIGURE 3.27 (a) SEM image of a tungsten filament magnified approximately 130 $\times$. (b) Result of global histogram equalization. (c) Image enhanced using local histogram statistics. (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

$$g(x, y) = \begin{cases} E g f(x, y), & \text{if } m_{s_y} \leq k_0 m_G \text{ and } k_1 \sigma_G \leq \sigma_{s_y} \leq k_2 \sigma_G \\ f(x, y), & \text{otherwise} \end{cases}$$

m_G : global mean; σ_G : global standard deviation

$k_0 = 0.4$; $k_1 = 0.02$; $k_2 = 0.4$; $E = 4$