107484 – Controle de Processos

Aula: Sintonia de Controladores PID

Síntese Direta e IMC

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Estrutura do Controlador

- Estrutura adotada para o controlador PID:

$$G_c(s) = K_P\left(1 + \frac{1}{sT_I} + T_D s\right)$$

- Relação de equivalência [Seborg, 2010]:

Parallel Form	Series Form
$G_c(s) = K_c\left(1 + \frac{1}{\tau_I s} + \tau_D s\right)$	$G_c(s) = K'_c\left(1 + \frac{1}{\tau'_I s}\right)(1 + \tau'_D s)$ †
$K_c \overset{*}{=} K'_c\left(1 + \frac{\tau'_D}{\tau'_I}\right)$	$K'_c = \frac{K_c}{2}\left(1 + \sqrt{1 - 4\tau_D/\tau_I}\right)$
$\tau_I = \tau'_I + \tau'_D$	$\tau'_I = \frac{\tau_I}{2}\left(1 + \sqrt{1 - 4\tau_D/\tau_I}\right)$
$\tau_D = \frac{\tau'_D \tau'_I}{\tau'_I + \tau'_D}$	$\tau'_D = \frac{\tau_I}{2}\left(1 - \sqrt{1 - 4\tau_D/\tau_I}\right)$

†These conversion equations are only valid if $\tau_D/\tau_I \leq 0.25$.

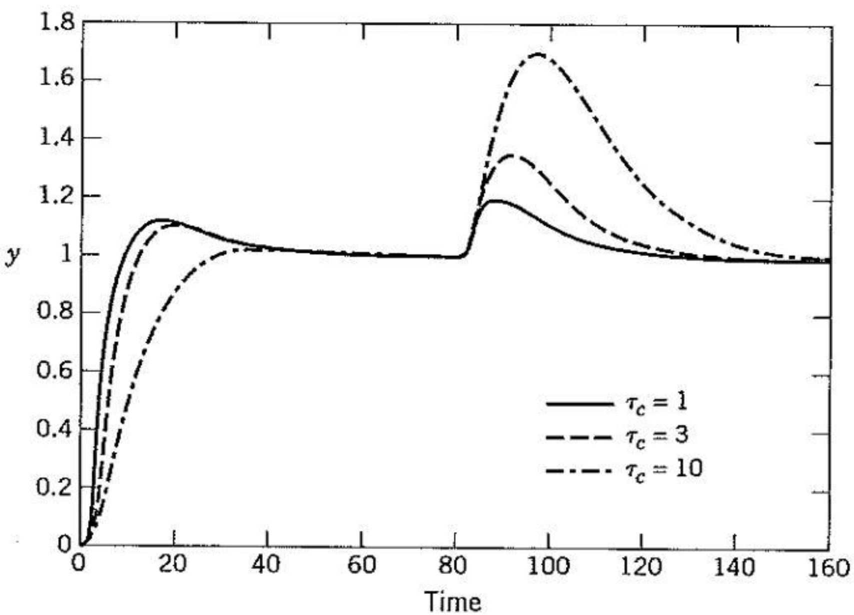
Método da Síntese Direta (SD)

Exemplo: Seja o processo e dinâmica desejada

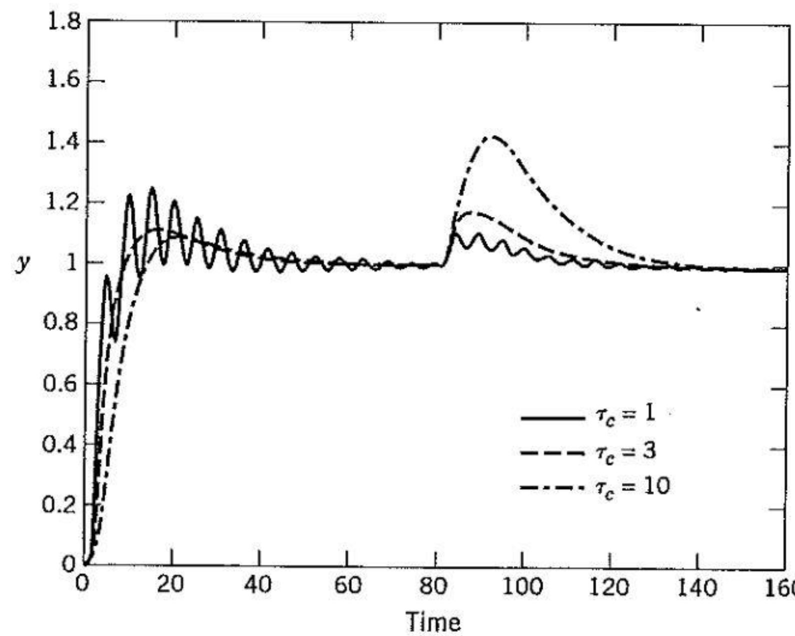
$$G_p(s) = \frac{2e^{-s}}{(10s+1)(5s+1)}$$

$$G_{MF}(s) = \frac{1e^{-s}}{\tau_c s + 1}$$

	$\tau_c = 1$	$\tau_c = 3$	$\tau_c = 10$
$K_c\ (\widetilde{K} = 2)$	3.75	1.88	0.682
$K_c\ (\widetilde{K} = 0.9)$	8.33	4.17	1.51
τ_I	15	15	15
τ_D	3.33	3.33	3.33



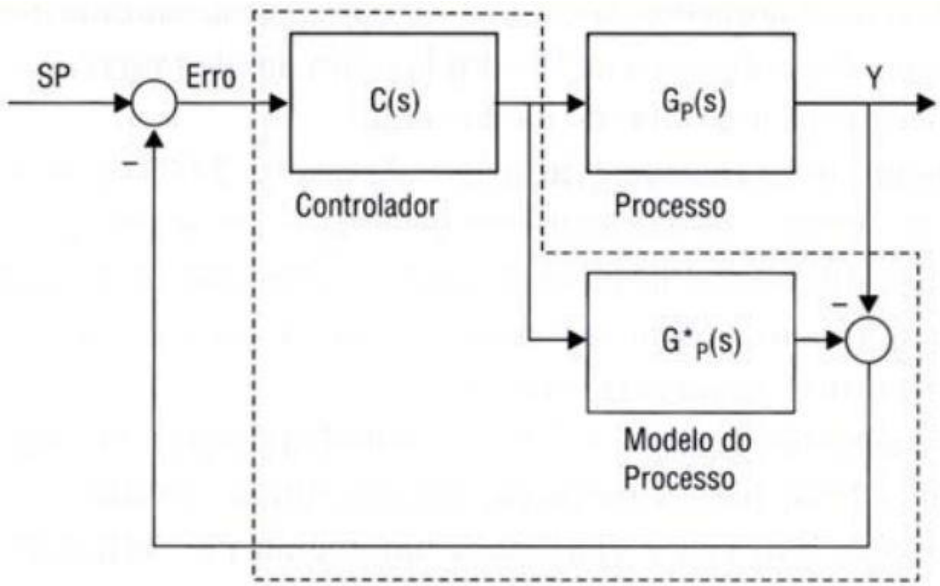
Resposta a SP e D (K correto).



Resposta a SP e D (K incorreto).

Método do Modelo Interno (IMC)

- Proposto por [Garcia e Morari, 1982 e Rivera et al., 1986]
- IMC e SD produzem os mesmo controladores (dinâmica precisa)
- IMC ⇨ permite incerteza de modelo, (robustez x desempenho)
- Modelo + especificação ⇨ controlador
- Uso de um modelo interno ⇨ fase de projeto e/ou operação



Método do Modelo Interno (IMC)

- Filtro ⇒ diminuir a sensibilidade a erros de modelagem
- IMC ⇒ funciona melhor para servo do que p/ reg.
- Escolha das ctes. tempo da malha fechada: $\lambda\ ou\ \tau_F$
  - [Astrom] $\lambda' = \lambda\tau \in [0.5\ 5], \lambda' < 1\ (\tau_{MF} < \tau_{MA})$
  - [Campos e Teixeira, 06] $\lambda = \tau_{dominante}$
  - [Chien e Fruehauf, 90] $\theta < \lambda < \tau$
  - [Skogestad, 03]$\lambda = \theta$
- Sintonia-λ é um exemplo de IMC desenvolvido usando a técnica de SD.

Método do Modelo Interno (IMC)

Desejando-se $G_{MF}(s) = \frac{1}{\lambda s+1}$ $(\lambda \geq 3\tau, \lambda \gg \theta)$ (sintonia Lambda):

Modelo do Processo	K_P	T_I	T_D
$\frac{K}{\tau s+1}$	$\frac{\tau}{K \times \lambda}$	τ	–
$\frac{K}{(\tau_1 s+1)(\tau_2 s+1)}$	$\frac{(\tau_1+\tau_2)}{K \times \lambda}$	$(\tau_1+\tau_2)$	$\frac{\tau_1 \times \tau_2}{(\tau_1+\tau_2)}$
$\frac{K}{\tau^2 s^2 + 2\xi\tau s + 1}$	$\frac{2\xi\tau}{K \times \lambda}$	$2\xi\tau$	$\frac{\tau}{2\xi}$
$\frac{K}{s}$	$\frac{1}{K \times \lambda}$	—	—
$\frac{K}{s(\tau s+1)}$	$\frac{1}{K \times \lambda}$	—	τ

Quando $G_P(s) = \frac{Ke^{-\theta s}}{\tau s+1}$ e $G_{MF}(s) = \frac{e^{-\theta s}}{\lambda s+1}$ [Rivera et al., 1986]

Controlador	K_P	T_I	T_D	Sugestão para o Desempenho
PID	$\frac{2\tau+\theta}{K \times (2\lambda+\theta)}$	$\tau + \left(\frac{\theta}{2}\right)$	$\frac{\tau \times \theta}{(2\tau+\theta)}$	$\frac{\lambda}{\theta} > 0.8$
PI	$\frac{(2\tau+\theta)}{K \times 2\lambda}$	$\tau + \left(\frac{\theta}{2}\right)$	—	$\frac{\lambda}{\theta} > 1.7$

[Rivera et al., 1986]

Método do Modelo Interno (IMC)

- [Luyben, 2001] PID Série com filtro no termo derivativo

$$U(s) = \mathrm{K_P}\left(1 + \frac{1}{sT_I}\right) \times \left(SP(s) - \frac{T_D s + 1}{\tau_F s + 1} Y(s)\right)$$

- Sugestão: $\lambda = \max\{0.25 \times \theta, 0.2 \times \tau\}$
- para processos FOPDT

$$G_P(s) = \frac{K e^{-\theta s}}{\tau s + 1}$$

- Sintonia proposta para PID e filtro

$$K_P = \frac{1}{K} \times \left(\frac{2\tau+\theta}{2(\lambda+\theta)}\right), T_I = \tau + \frac{\theta}{2}, T_D = \frac{\tau\theta}{2\tau+\theta} \text{ e } \tau_F = \frac{\lambda\theta}{2(\lambda+\theta)}$$

Método do Modelo Interno (IMC)

- [Skogestad, 2004] PID Série (derivativo na PV)

$$U(s) = \mathrm{K_P}\left(1 + \frac{1}{sT_I}\right) \times \left(SP(s) - \frac{T_D s + 1}{\tau_F s + 1} Y(s)\right)$$

$\tau_F = 0.01\ T_D$ (usual) ou $\tau_F = 0.01\ T_D$ (processos ruidosos)

Resposta ideal em MF

$$\frac{Y(s)}{SP(s)} = \frac{1}{\lambda s + 1} e^{-\theta s} \text{ (atraso } \theta \text{ inevitável)}$$

- Sugestão: $\lambda = \theta$ (compromisso robustez e desempenho)
- Se o desempenho não estiver adequado ➡ aumentar λ
- Para processos ruidosos ➡ (1) aumentar τ_F até $\tau_F = {}^{\theta}/_{2}$; (2) eliminar T_D; (3) aumentar λ

Método do Modelo Interno (IMC)

$$G_{MF}(s) = \frac{1}{\lambda s + 1} e^{-\theta s}$$

PID indicado quando
$\theta < \tau_2 < \tau_1$

Modelo do Processo	K_P	T_I	T_D
$\frac{K}{\tau s + 1} e^{-\theta s}$	$\frac{\tau}{K \times (\lambda + \theta)}$	$\min\{\tau, 4 \times (\lambda + \theta)\}$	—
$\frac{K}{(\tau_1 s + 1)(\tau_2 s + 1)} e^{-\theta s}$	$\frac{\tau_1}{K \times (\lambda + \theta)}$	$\min\{\tau_1, 4 \times (\lambda + \theta)\}$	τ_2
$K \times e^{-\theta s}$	$\frac{1}{K}$	$\lambda + \theta$	—
$\frac{K}{s} e^{-\theta s}$	$\frac{1}{K \times (\lambda + \theta)}$	$4 \times (\lambda + \theta)$	—
$\frac{K}{s(\tau_2 s + 1)} e^{-\theta s}$	$\frac{1}{K \times (\lambda + \theta)}$	$4 \times (\lambda + \theta)$	τ_2

[Skogestad, Simple analytic rules for model reduction and PID controller tuning, 2003]

Escolha de filtro passa baixa no método IMC que leva a equivalência com Síntese Direta com especificação:

$$G_{MF}(s) = \frac{1}{\tau_c s + 1}$$

para processos com atraso:

$$G_{MF}(s) = \frac{1}{\tau_c s + 1} e^{-\theta s}$$

Controlador:

$$G_c(s) = K_c \left(1 + \frac{1}{s\tau_I} + \tau_D s\right)$$

Aproximação do atraso:

$$e^{-\theta s} \cong 1 - \theta s$$

(casos G, I, J, K, L, M, O)

$$e^{-\theta s} \cong \frac{1 - \theta/2s}{1 + \theta/2s}$$

(casos H, N)

[Seborg, 2010]

Table 12.1 IMC-Based PID Controller Settings for $G_c(s)$ (Chien and Fruehauf, 1990)

Case	Model	K_cK	τ_I	τ_D
A	$\frac{K}{\tau s + 1}$	$\frac{\tau}{\tau_c}$	τ	—
B	$\frac{K}{(\tau_1 s + 1)(\tau_2 s + 1)}$	$\frac{\tau_1 + \tau_2}{\tau_c}$	$\tau_1 + \tau_2$	$\frac{\tau_1\tau_2}{\tau_1 + \tau_2}$
C	$\frac{K}{\tau^2 s^2 + 2\zeta\tau s + 1}$	$\frac{2\zeta\tau}{\tau_c}$	$2\zeta\tau$	$\frac{\tau}{2\zeta}$
D	$\frac{K(-\beta s + 1)}{\tau^2 s^2 + 2\zeta\tau s + 1}$, $\beta > 0$	$\frac{2\zeta\tau}{\tau_c + \beta}$	$2\zeta\tau$	$\frac{\tau}{2\zeta}$
E	$\frac{K}{s}$	$\frac{2}{\tau_c}$	$2\tau_c$	—
F	$\frac{K}{s(\tau s + 1)}$	$\frac{2\tau_c + \tau}{\tau_c^2}$	$2\tau_c + \tau$	$\frac{2\tau_c\tau}{2\tau_c + \tau}$
G	$\frac{Ke^{-\theta s}}{\tau s + 1}$	$\frac{\tau}{\tau_c + \theta}$	τ	—
H	$\frac{Ke^{-\theta s}}{\tau s + 1}$	$\frac{\tau + \frac{\theta}{2}}{\tau_c + \frac{\theta}{2}}$	$\tau + \frac{\theta}{2}$	$\frac{\tau\theta}{2\tau + \theta}$
I	$\frac{K(\tau_3 s + 1)e^{-\theta s}}{(\tau_1 s + 1)(\tau_2 s + 1)}$	$\frac{\tau_1 + \tau_2 - \tau_3}{\tau_c + \theta}$	$\tau_1 + \tau_2 - \tau_3$	$\frac{\tau_1\tau_2 - (\tau_1 + \tau_2 - \tau_3)\tau_3}{\tau_1 + \tau_2 - \tau_3}$
J	$\frac{K(\tau_3 s + 1)e^{-\theta s}}{\tau^2 s^2 + 2\zeta\tau s + 1}$	$\frac{2\zeta\tau - \tau_3}{\tau_c + \theta}$	$2\zeta\tau - \tau_3$	$\frac{\tau^2 - (2\zeta\tau - \tau_3)\tau_3}{2\zeta\tau - \tau_3}$
K	$\frac{K(-\tau_3 s + 1)e^{-\theta s}}{(\tau_1 s + 1)(\tau_2 s + 1)}$	$\frac{\tau_1 + \tau_2 + \frac{\tau_3\theta}{\tau_c + \tau_3 + \theta}}{\tau_c + \tau_3 + \theta}$	$\tau_1 + \tau_2 + \frac{\tau_3\theta}{\tau_c + \tau_3 + \theta}$	$\frac{\tau_3\theta}{\tau_c + \tau_3 + \theta} + \frac{\tau_1\tau_2}{\tau_1 + \tau_2 + \frac{\tau_3\theta}{\tau_c + \tau_3 + \theta}}$
L	$\frac{K(-\tau_3 s + 1)e^{-\theta s}}{\tau^2 s^2 + 2\zeta\tau s + 1}$	$\frac{2\zeta\tau + \frac{\tau_3\theta}{\tau_c + \tau_3 + \theta}}{\tau_c + \tau_e + \theta}$	$2\zeta\tau + \frac{\tau_3\theta}{\tau_c + \tau_3 + \theta}$	$\frac{\tau_3\theta}{\tau_c + \tau_3 + \theta} + \frac{\tau^2}{2\zeta\tau + \frac{\tau_3\theta}{\tau_c + \tau_3 + \theta}}$
M	$\frac{Ke^{-\theta s}}{s}$	$\frac{2\tau_c + \theta}{(\tau_c + \theta)^2}$	$2\tau_c + \theta$	—
N	$\frac{Ke^{-\theta s}}{s}$	$\frac{2\tau_c + \theta}{\left(\tau_c + \frac{\theta}{2}\right)^2}$	$2\tau_c + \theta$	$\frac{\tau_c\theta + \frac{\theta^2}{4}}{2\tau_c + \theta}$
O	$\frac{Ke^{-\theta s}}{s(\tau s + 1)}$	$\frac{2\tau_c + \tau + \theta}{(\tau_c + \theta)^2}$	$2\tau_c + \tau + \theta$	$\frac{(2\tau_c + \theta)\tau}{2\tau_c + \tau + \theta}$

Bibliografia

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- Dale E. Seborg, Duncan A. Mellichamp, Thomas F. Edgar e Francis J. Doyle, *Process Dynamics and Control*, 3ª ed., 2010, Wiley.